Economic Theory II
Lecture 7 [1]

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[1] Content of current lecture slides derives on slides prepared by Dr Valentin Jouvanceau previously.

The Production side [2]

- We analyse the microeconomics of the firm problem.
- We derive expressions for labour and investment demand.
- We also augment households to choose labour supply.

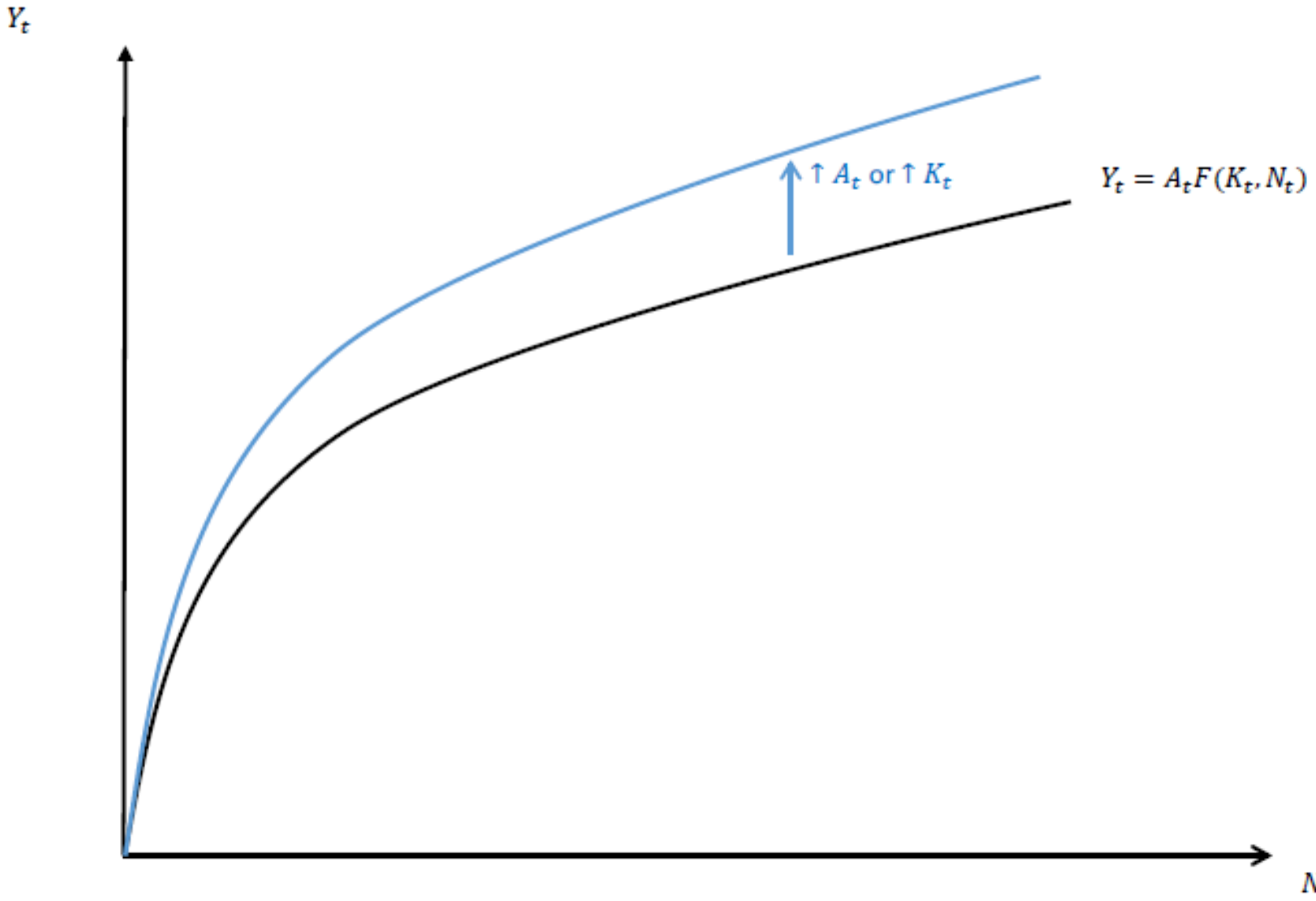
[2] Chapter 12 of GLS.

Model set-up

- Firms produce using the standard production function we have seen so far: $Y_t = A_t F(K_t, N_t)$.
- Two changes: A_t time varying, and $Z = 1$ (not shown in above equation).
- Properties same as before:
  - Diminishing marginal products for both factors.
  - Cross-partial derivatives positive.
  - Both inputs necessary for production.
  - Constant Returns to Scale.
- Now we no longer use 'per capita' variables. Why?

Graphical Analysis

The Production Function (Cobb-Douglas):



Model set-up

- Separation of ownership and management for firms: Firms own capital stock.
- Capital evolves as: $K_{t+1} = I_t + (1 - \delta)K_t$.
- Firms borrow funds from a financial intermediary at rate r_t.
- This rate is same for households as well.
- Investment funded fully by borrowing: $I_t = B_t^I$. Realistic?

Dividends

- Difference between revenues and costs is paid out as dividends: $D_t = Y_t - w_t N_t$. Why no R^K?
- Model ends in second period, so what happens to capital? $I_{t+1} = -(1-\delta)K_{t+1}$.
- What about loan with interest? Adjusted in dividends.
- Dividends in second period:

$$D_{t+1} = Y_{t+1} - w_{t+1}N_{t+1} + (1-\delta)K_{t+1} - (1+r_t)B_t^I$$

- What is the above in terms of I?

Firm Value

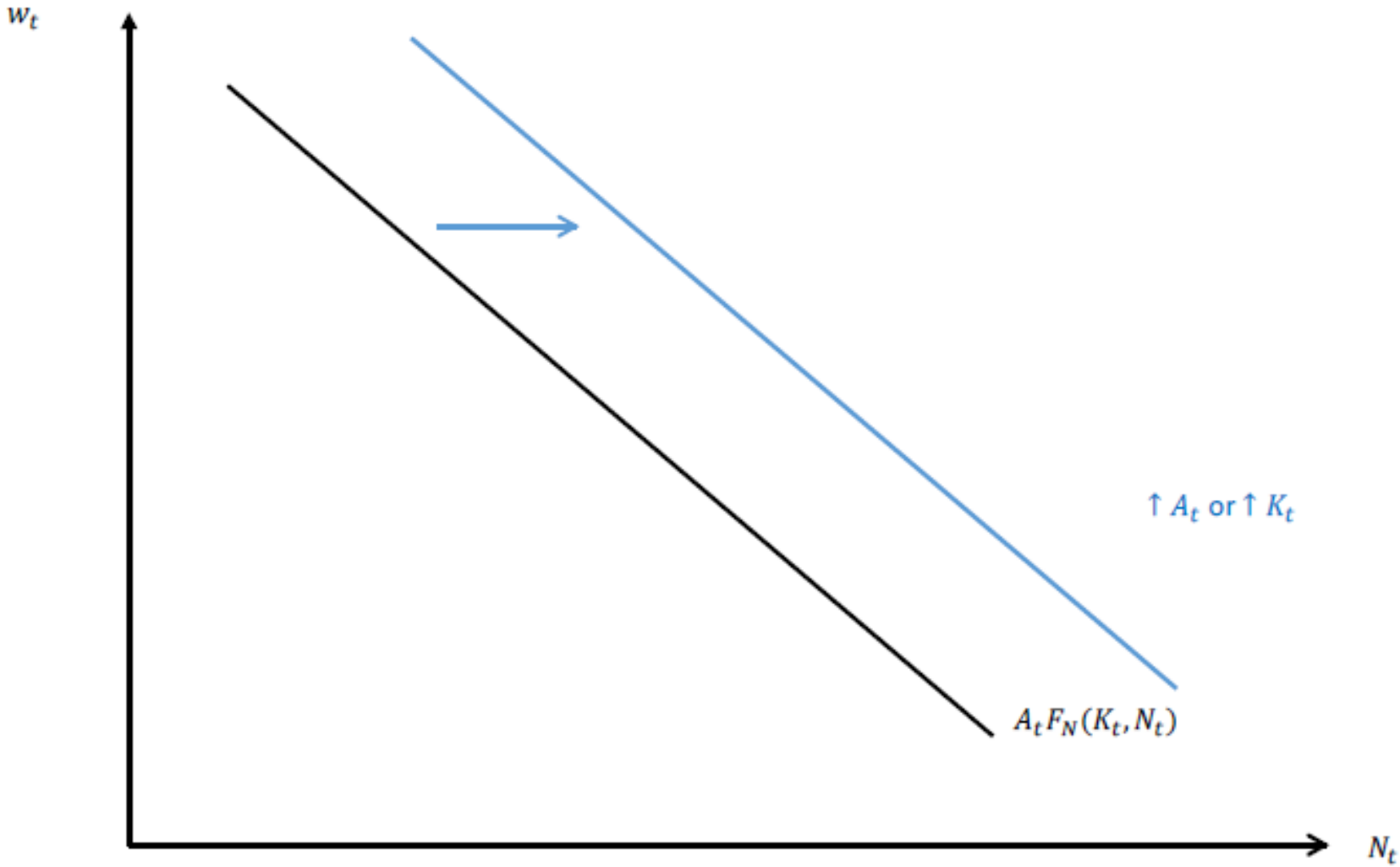
- Value of the firm today is the sum of: 1. Dividends today, 2. Dividends in future, discounted:

$$V_t = D_t + \frac{D_{t+1}}{1 + r_t}$$

- Why discounted with $(1 + r_t)$? What is the intuition?
- Why is this the value of the firm?
- Maximisation problem after substituting for Y_t, K_{t+1}, and B_t^I?
- Choose N_t and K_{t+1} to maximise value. Why?

Labour Demand

$$w_t = A_t F_N(K_t, N_t) \Rightarrow N_t = N^d(\underset{-}{w_t}, \underset{+}{A_t}, \underset{+}{K_t}).$$

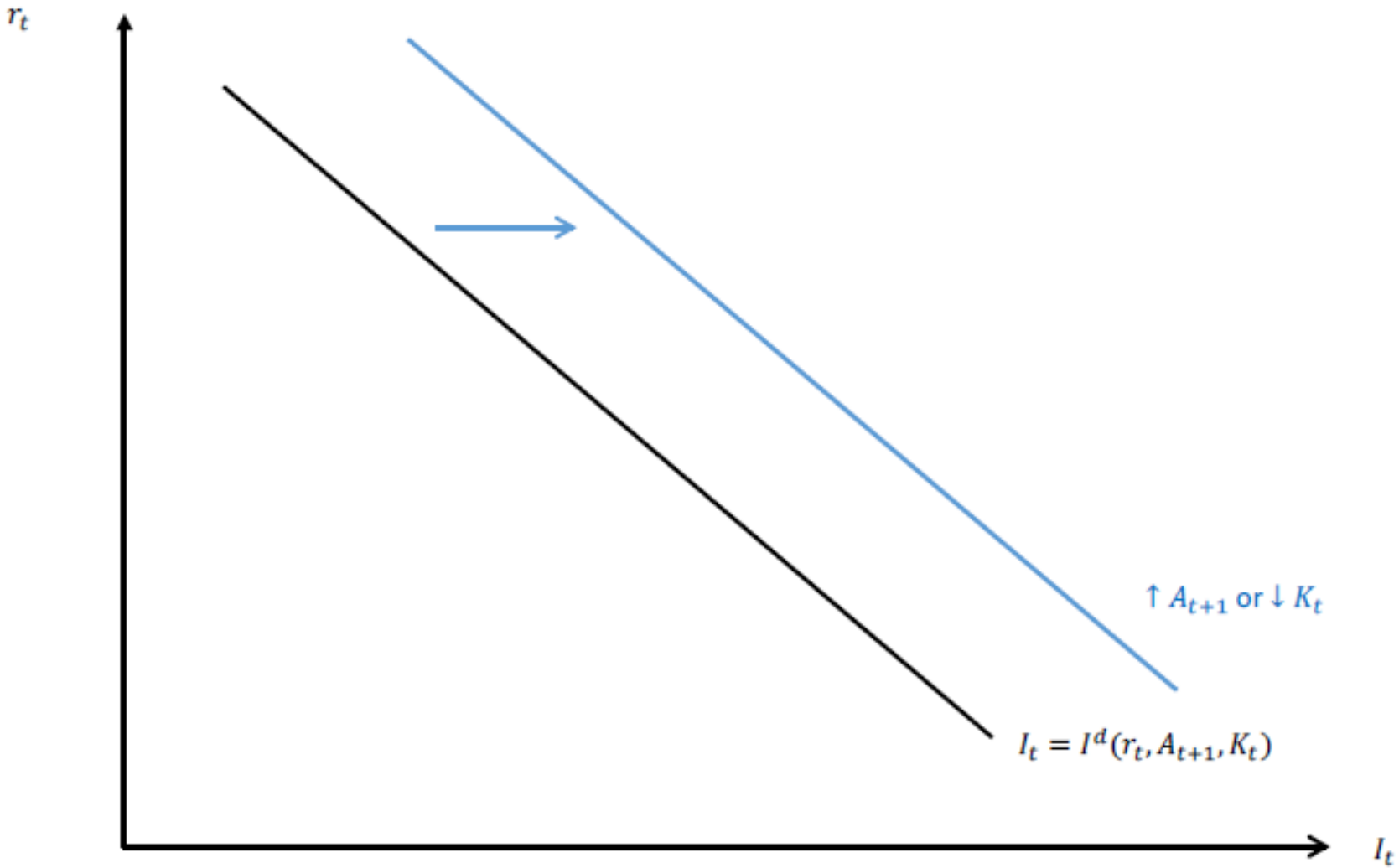


Investment Demand

- $(1 + r_t) = A_{t+1} F_K(K_{t+1}, N_{t+1}) + (1 - \delta) \Rightarrow K_{t+1} = K^d(\underset{-}{r_t}, \underset{+}{A_{t+1}})$: What is the intuition? What if it doesn't hold?
- What about Investment? Impact of r_t and A_{t+1}, given K_t? Same as K_{t+1}?
- What about impact of K_t on investment I_t? Because we choose K_{t+1} and I_t is the amount needed to reach that, a high K_t will reduce the needed I_t.
- $I_t = I^d(\underset{-}{r_t}, \underset{+}{A_{t+1}}, \underset{-}{K_t})$.

Investment Demand

$$I_t = I^d(\underset{-}{r_t}, \underset{+}{A_{t+1}}, \underset{-}{K_t})$$



Households

- Mostly same as before.
- Only difference is that now they choose labour supply, in addition to consumption.
- Normalise total time available to 1: Labour N_t + Leisure $L_t = 1 \Rightarrow L_t = 1 - N_t$.
- Households choose leisure because that gives them utility, indirectly choosing labour supply.
- $u(C_t, L_t) \Rightarrow u(C_t, 1 - N_t)$: Choose C and N.

Households

- Properties:
  - Diminishing marginal utility of consumption: $u_C > 0$ and $u_{CC} < 0$.
  - Diminishing marginal utility of leisure: $u_L > 0 \Leftrightarrow u_N < 0$ and $u_{LL} < 0 \Leftrightarrow u_{NN} > 0$.
  - Cross-partial u_{CL} depends on specific functional form of utility function.
- Lifetime utility:

$$U = u(C_t, 1 - N_t) + \beta u(C_{t+1}, 1 - N_{t+1})$$

Households

- Period budget constraints:
  - $C_t + S_t \leq w_t N_t + D_t$.
  - $C_{t+1} \leq w_{t+1} N_{t+1} + D_{t+1} + (1 + r_t) S_t + D^I_{t+1}$.
- D^I_{t+1} is dividend payout from the financial intermediary (discussed later).
- Lifetime budget constraint:

$$C_t + \frac{C_{t+1}}{1 + r_t} = w_t N_t + D_t + \frac{w_{t+1} N_{t+1} + D_{t+1} + D^I_{t+1}}{1 + r_t}$$

Households

Optimisation problem:

$$\max_{C_t, C_{t+1}, N_t, N_{t+1}} U = u(C_t, 1 - N_t) + \beta u(C_{t+1}, 1 - N_{t+1}) \tag{1}$$

subject to:

$$C_t + \frac{C_{t+1}}{1 + r_t} = w_t N_t + D_t + \frac{w_{t+1} N_{t+1} + D_{t+1} + D^I_{t+1}}{1 + r_t} \tag{2}$$

Why can't we just choose savings like before?

Households

Can simplify optimisation by reducing one variable. Use (2) to get C_{t+1} in terms of C_t:

$$C_{t+1} = (1 + r_t)(w_t N_t + D_t - C_t) + w_{t+1} N_{t+1} + D_{t+1} + D^I_{t+1} \tag{3}$$

Now, unconstrained optimisation:

$$\begin{aligned} \max_{C_t, N_t, N_{t+1}} \quad & U = u(C_t, 1 - N_t) \\ & + \beta u(\underbrace{(1 + r_t)(w_t N_t + D_t - C_t) + w_{t+1} N_{t+1} + D_{t+1} + D^I_{t+1}}_{C_{t+1}}, \\ & 1 - N_{t+1}) \end{aligned} \tag{4}$$

Households

- Optimality conditions:

$$u_C(C_t, 1 - N_t) = \beta(1 + r_t)u_C(C_{t+1}, 1 - N_{t+1}) \quad (5)$$
$$u_L(C_t, 1 - N_t) = \beta(1 + r_t)w_t u_C(C_{t+1}, 1 - N_{t+1}) \quad (6)$$
$$u_L(C_{t+1}, 1 - N_{t+1}) = w_{t+1} u_C(C_{t+1}, 1 - N_{t+1}) \quad (7)$$

- Combining (5) and (6),

$$u_L(C_t, 1 - N_t) = w_t u_C(C_t, 1 - N_t) \quad (8)$$

- What is the intuition for the Euler? What adjusts? Why?

Households

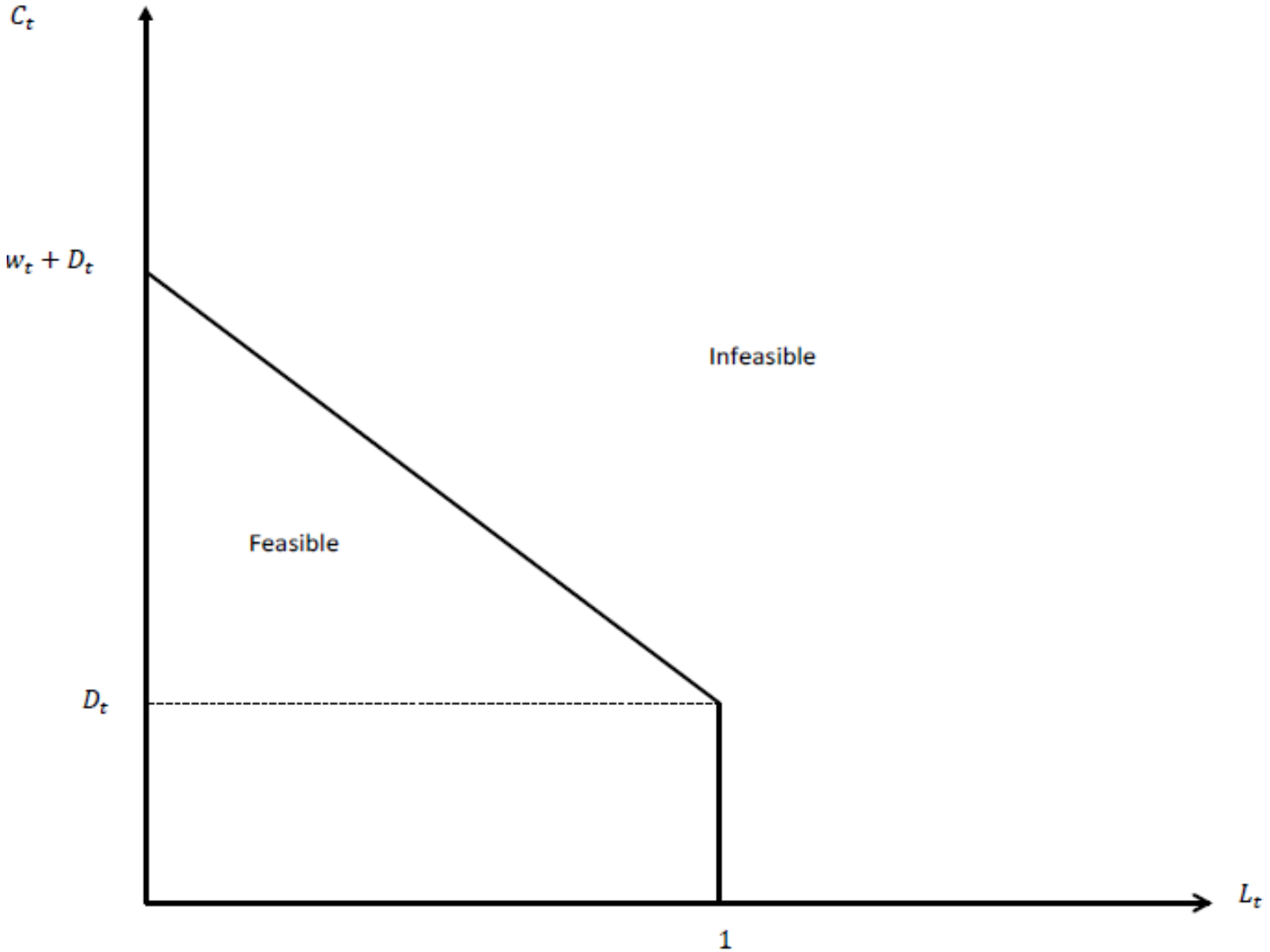
- 'Cost of *inter* temporal substitution' interpretation: $(1 + r_t)$ is the cost of 'buying' more consumption tomorrow. As demand for tomorrow's consumption rises, price rises too!
- Similar interpretation for (8): 'Cost of *intra* temporal substitution'.
- If $C_t \uparrow$, need to provide more labour $N_t \uparrow$ and leisure $L_t \downarrow$.
- Depending on form of utility function, price of this substitution w_t will change.
- One functional form to try is: $u(C_t, 1 - N_t) = \ln C_t + \theta_t \ln(1 - N_t)$: what does the consumption function look like?

Consumption-Leisure

- Consider only one period: $C_t = w_t N_t + D_t$.
- When $N_t = 0$, $L_t = 1$, and $C_t = D_t$.
- When $N_t = 1$, $L_t = 0$, and $C_t = w_t + D_t$.
- What if $D_t = 0$? What does graph look like?
- What happens when $D_t > 0$? What does graph look like?

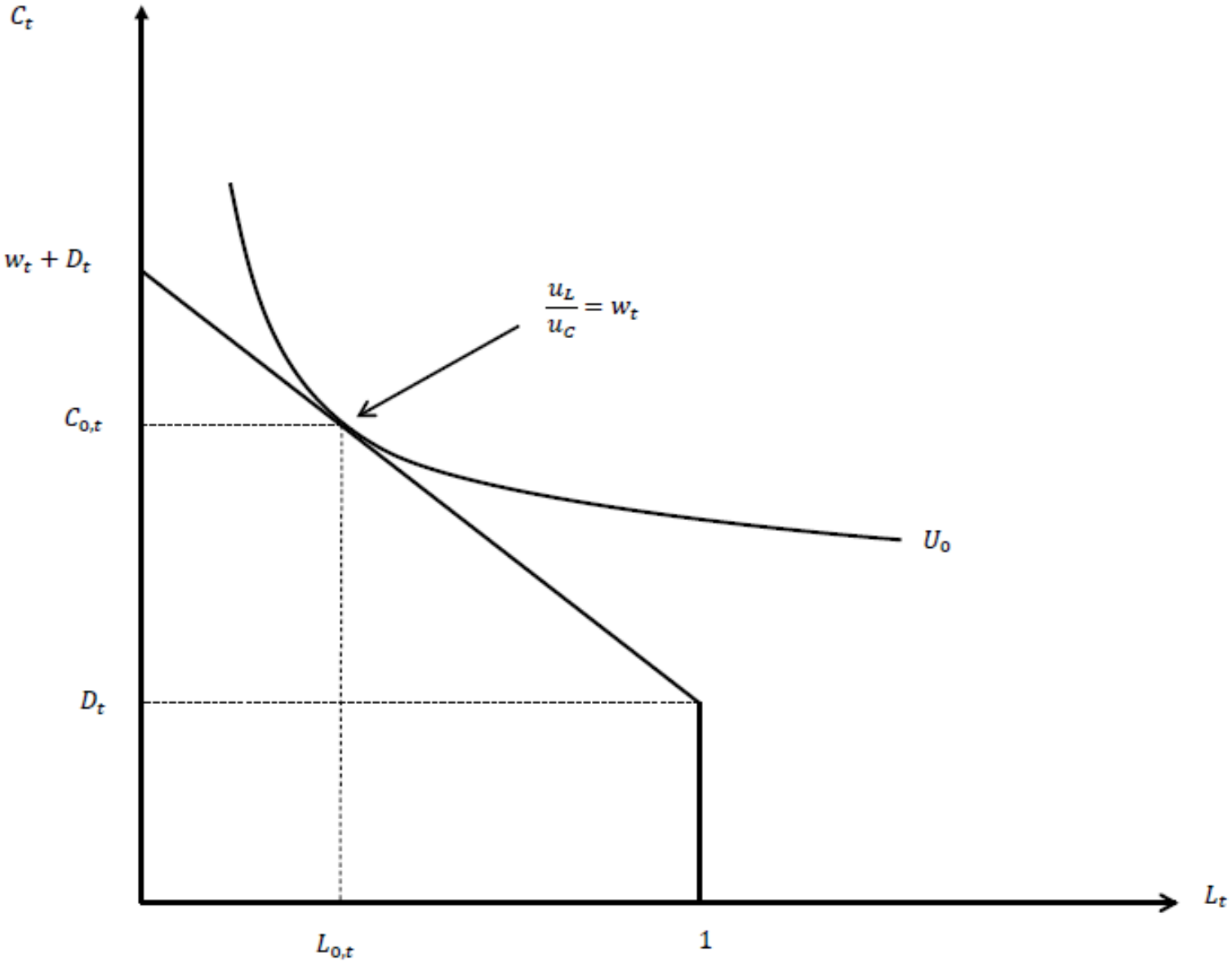
Graphical Analysis

Consumption-Leisure plot:



Graphical Analysis

Consumption-Leisure equilibrium:



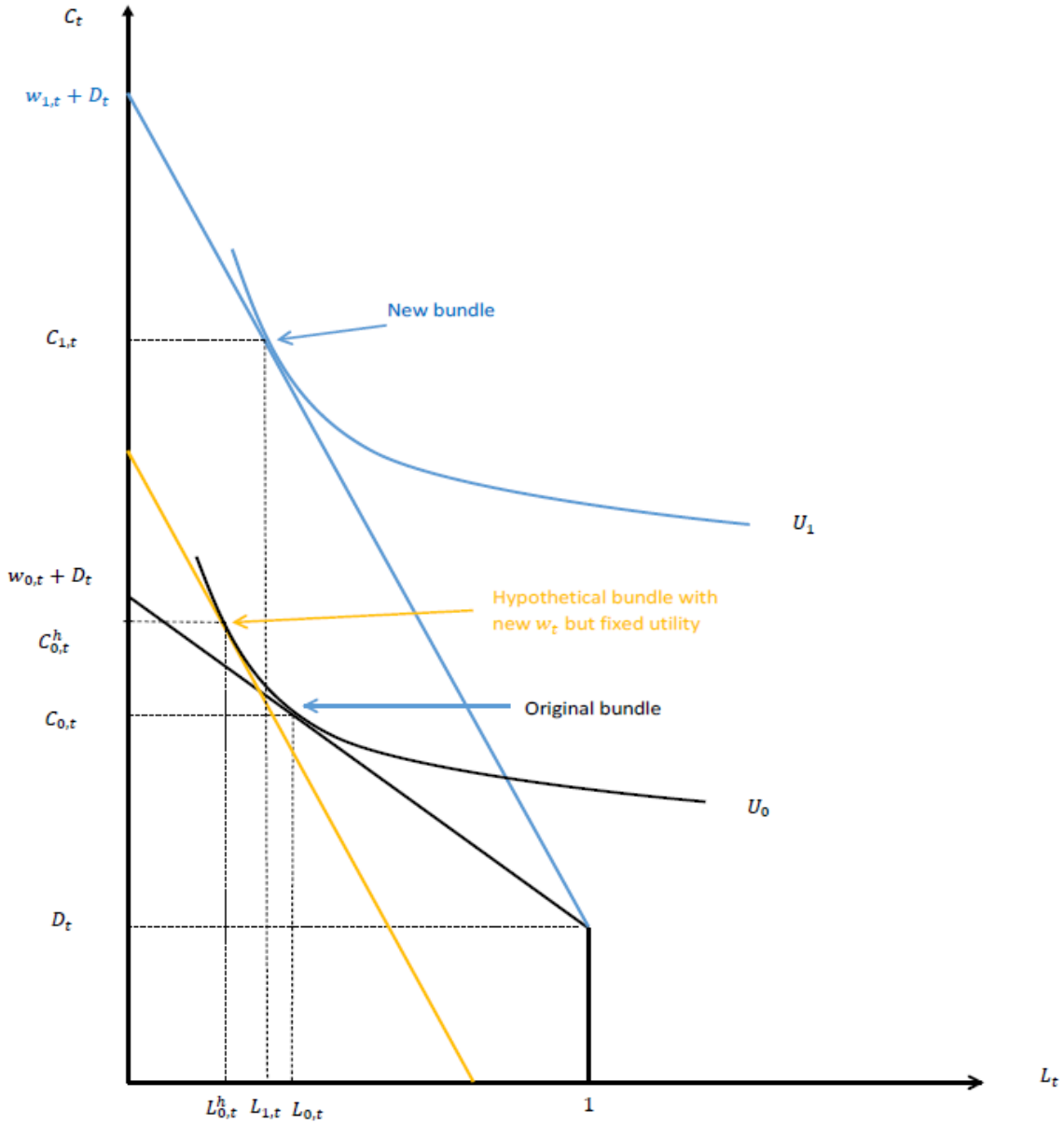
Graphical Analysis

Effect of increase in w_t:

- From (6), the 'price' of substituting between consumption and leisure is w_t.
- If $w_t \uparrow$, dual effect of substitution and income.
- Substitution effect: When $w_t \uparrow$, leisure becomes more costly, so substitute consumption for leisure $\Rightarrow$ consumption and labour supply both increase along the same indifference curve.
- Income effect: As $w_t \uparrow$, income of household $\uparrow$, so households feel richer and want to consume more and enjoy more leisure, so consumption rises and labour supply falls.
- Net effect of income and substitution clearly raises consumption, but ambiguous about labour supply.
- Textbook assumes substitution effect always dominates.

Graphical Analysis

Increase in w_t:

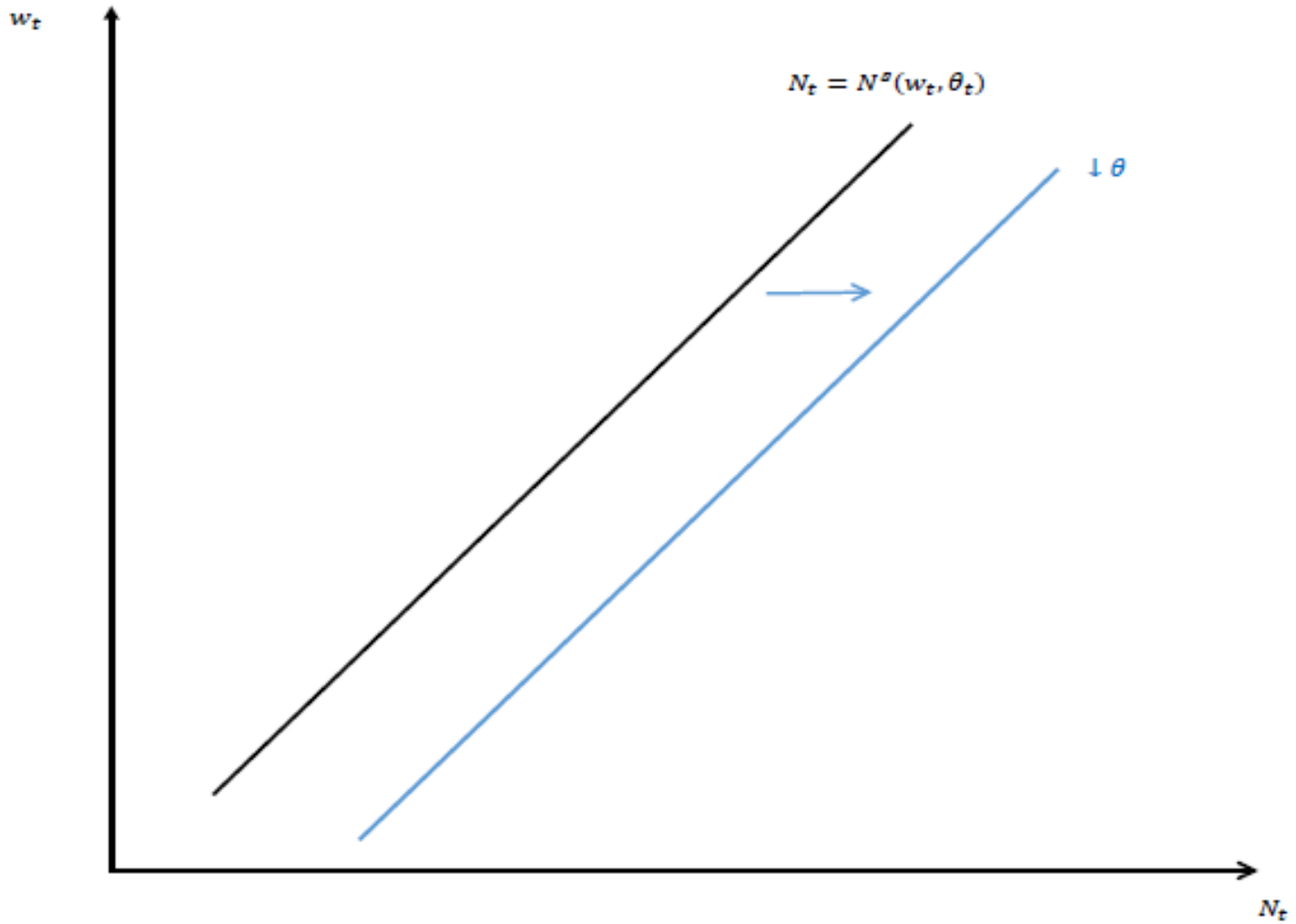


Labour Supply

- For the log specification: $w_t = \theta_t \frac{C_t}{1-N_t} = \theta_t \frac{C^d(Y_t, Y_{t+1}, r_t)}{1-N_t}$.
- In theory, all income $(Y_t + Y_{t+1})$ and the rate of interest (r_t), along with wage rate w_t and the exogenous variable θ_t, influence labour supply.
- However, we can assume specification of preferences such that C_t does not appear here, and $w_t = \theta_t \frac{1}{1-N_t}$.
- This gives the labour supply $N_t = N^s(\underset{+}{w_t}, \underset{-}{\theta_t})$.
- The impact coming from changes in income and in r_t through consumption can be considered negligible, and can be ignored.

Graphical Analysis

Increase in w_t:



Financial Intermediary

- Simple form of financial intermediary.
- Take savings S_t from households, and lend them to firms for investment $B_t^I = I_t$.
- Lending rate = borrowing rate.
- Profits $D_t^I = r_t B_t^I - r_t S_t$. Zero in equilibrium.
- Richer specification would be to have an interest rate spread to make profits, etc.

Equilibrium

"Set of prices and allocations, where all agents behave optimally and all markets clear."

- What does labour demand look like? What does labour market clearing look like?
- For the savings market: $S_t = B_t^I = I_t$.
- Above, combined with household's budget constraint and definition of dividends, gives goods market clearing: $Y_t = C_t + I_t$.
- Show the same holds for period 2...

Equilibrium

Full equilibrium characterised by:

$$C_t = C^d(Y_t, Y_{t+1}, r_t) \tag{9}$$
$$N_t = N^s(w_t, \theta_t) \tag{10}$$
$$N_t = N^d(w_t, A_t, K_t) \tag{11}$$
$$I_t = I^d(r_t, A_{t+1}, K_t) \tag{12}$$
$$Y_t = A_t F(K_t, N_t) \tag{13}$$
$$Y_t = C_t + I_t \tag{14}$$

6 equations for 6 endogenous variables:

$$C, \quad Y, \quad N, \quad K, \quad w, \quad r$$

Endogenous quantities: C, Y, N, K. Endogenous Prices: w, r. Exogenous variables: A, θ.

End