Economic Theory II
Lecture 3 [1]

Coordinator: Dr Benjamin Hemingway
Teacher: Anup Mulay
anup.mulay@evaf.vu.lt

[1] Content of current lecture slides derives on slides prepared by Dr Valentin Jouvanceau previously.

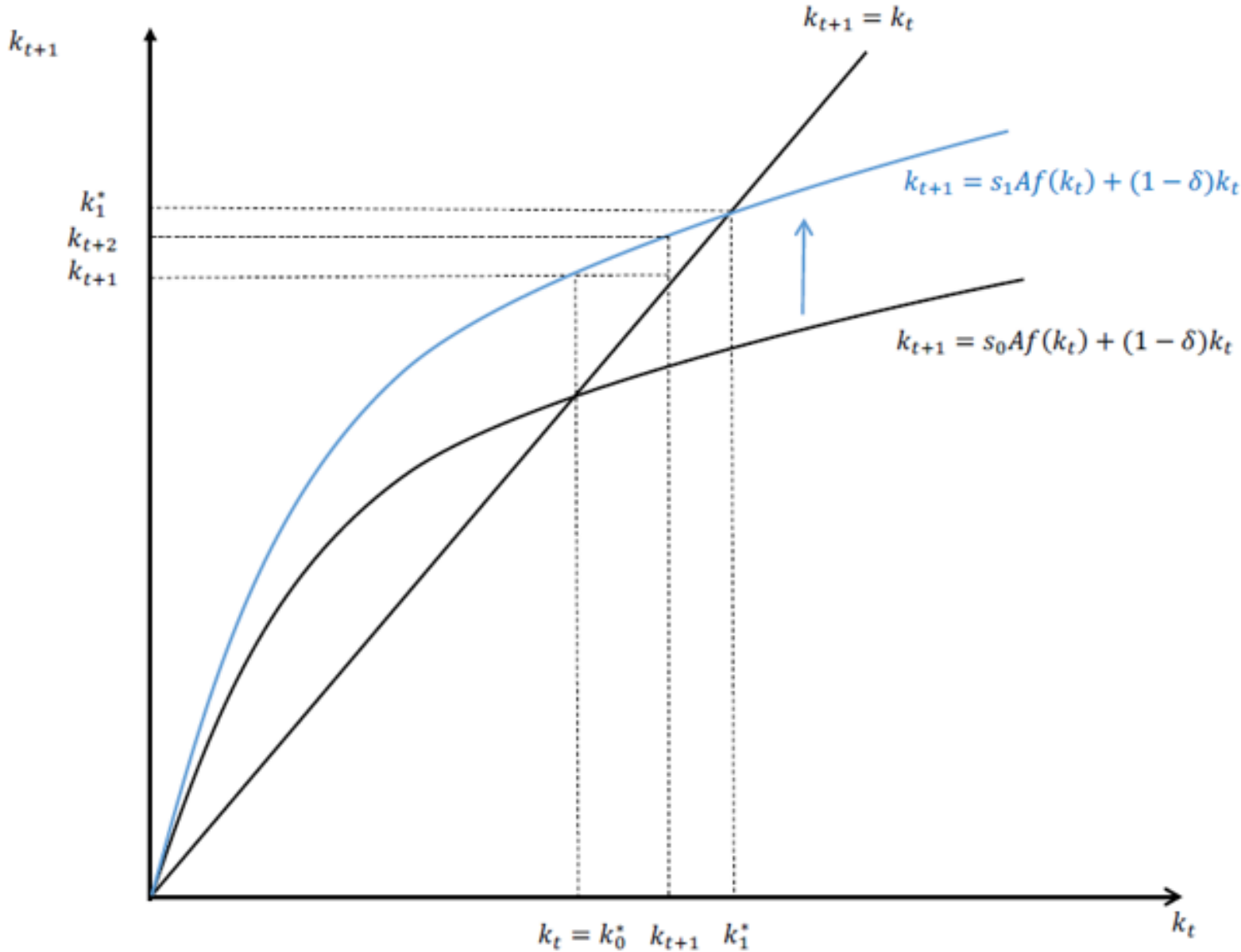
Experiments [2]

- We will check what happens to the model if the saving rate s or the productivity A changes permanently.
- Let's first change the saving rate s. If s increases:
  - Savings increase, so investment increases.
  - Capital stock increases, and settles at a new, higher steady state.
  - Consumption drops at the point when saving rate suddenly rises, then rises slowly to the new level in steady state.
  - Output and wages rise slowly to the new steady state level.
  - Rental rate of capital *falls*. Why?

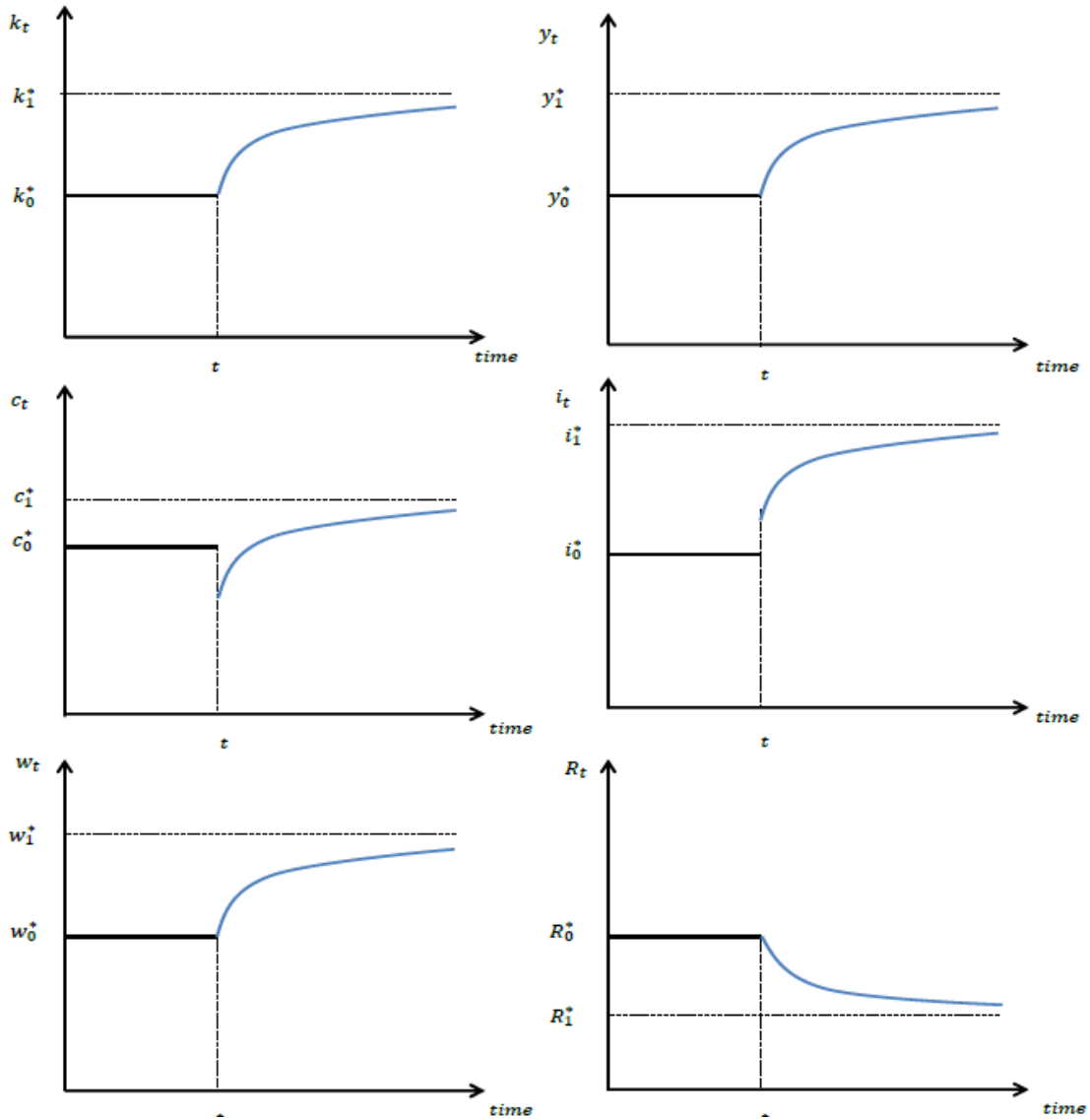
[2] Chapter 5 of GLS.

Increase in saving

Permanent increase in s:

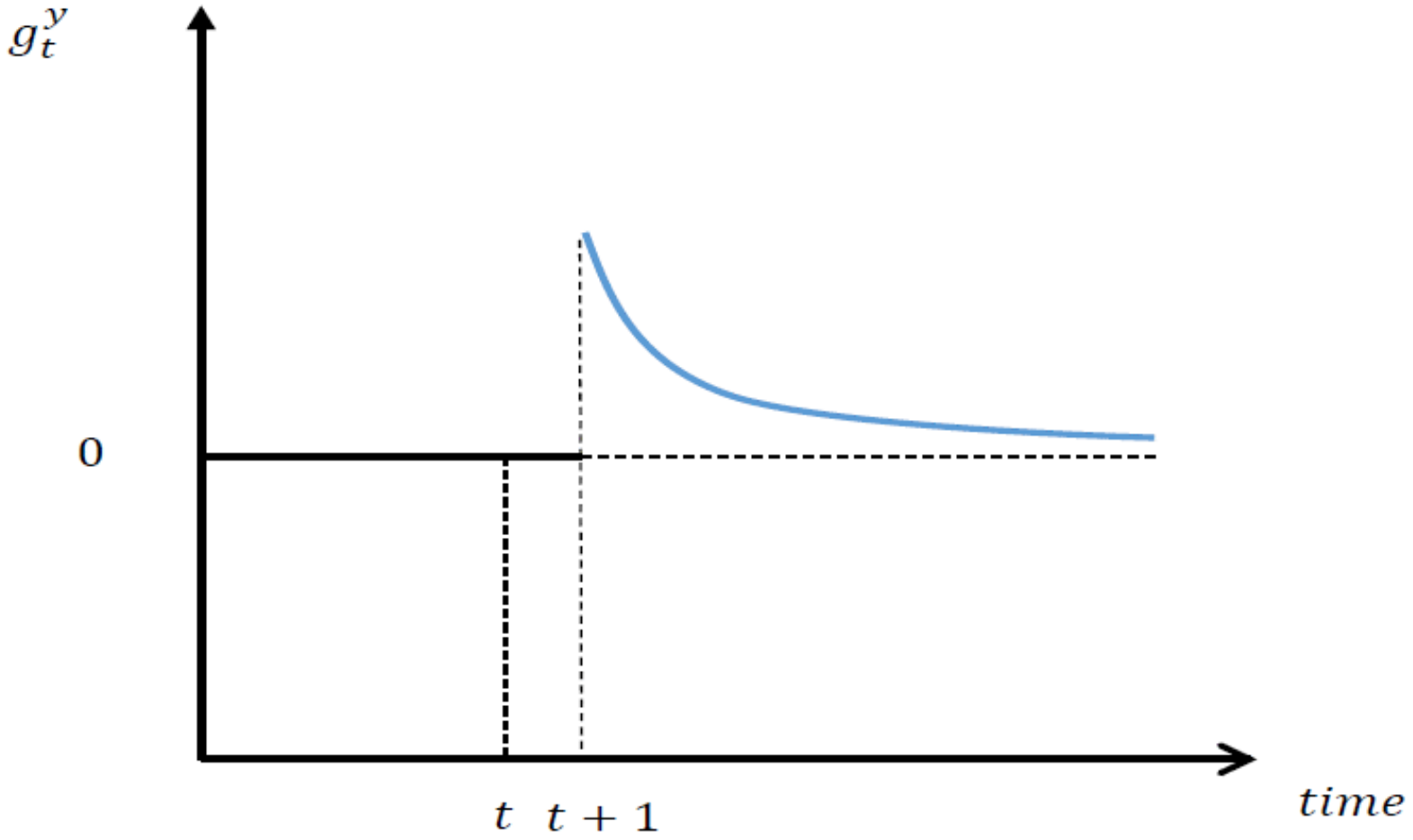


Increase in saving



Increase in saving

Dynamic path of output growth:



Increase in productivity

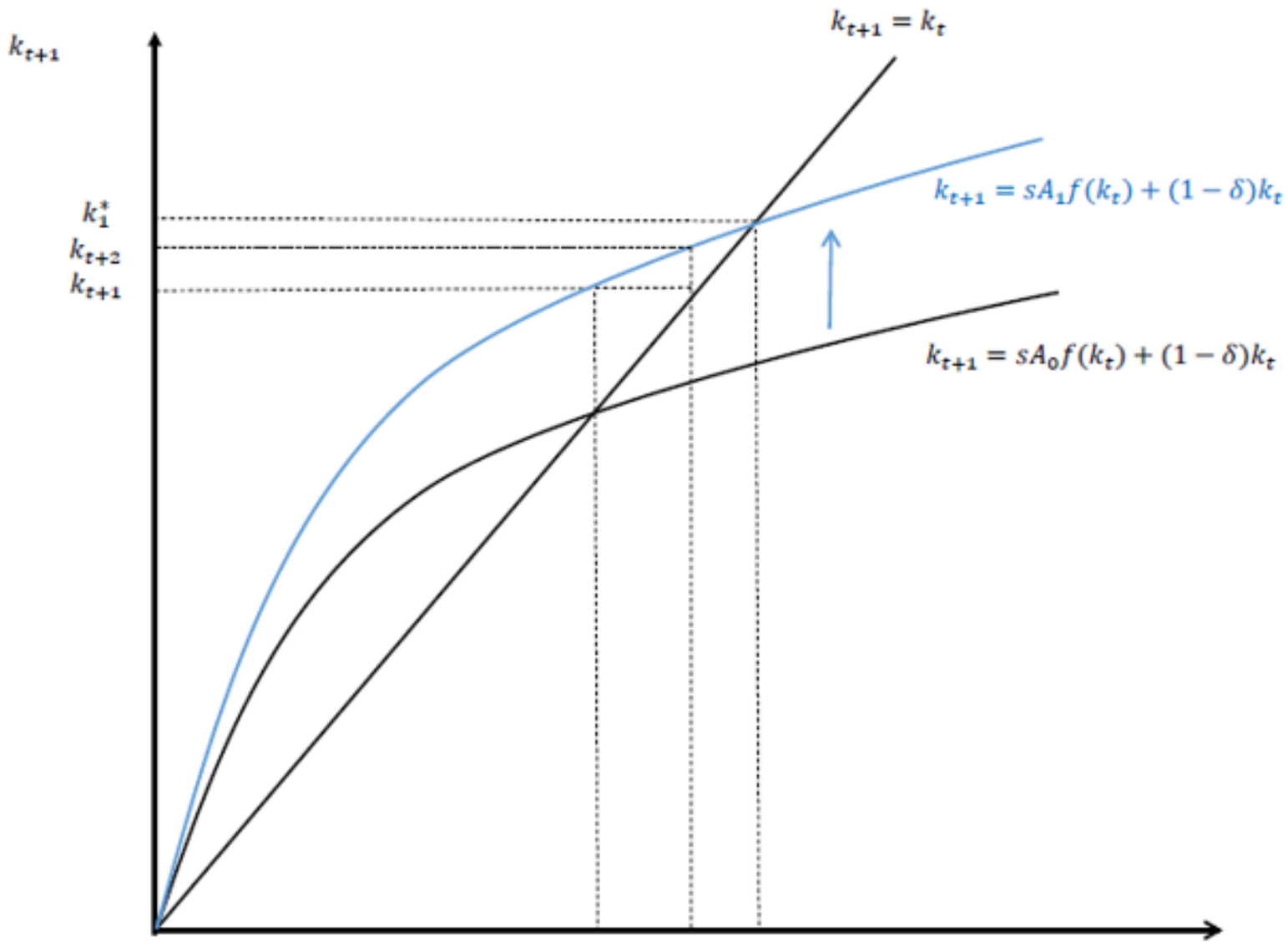
A permanent shift in A works as follows:

- A acts as a 'supply shock': Production suddenly goes up.
- As output jumps, consumption and investment also jump.
- As investment goes up, capital level also starts increasing.
- Factor prices also jump.
- Wages settle at new steady state level, but rent returns to the same steady state level. Why?

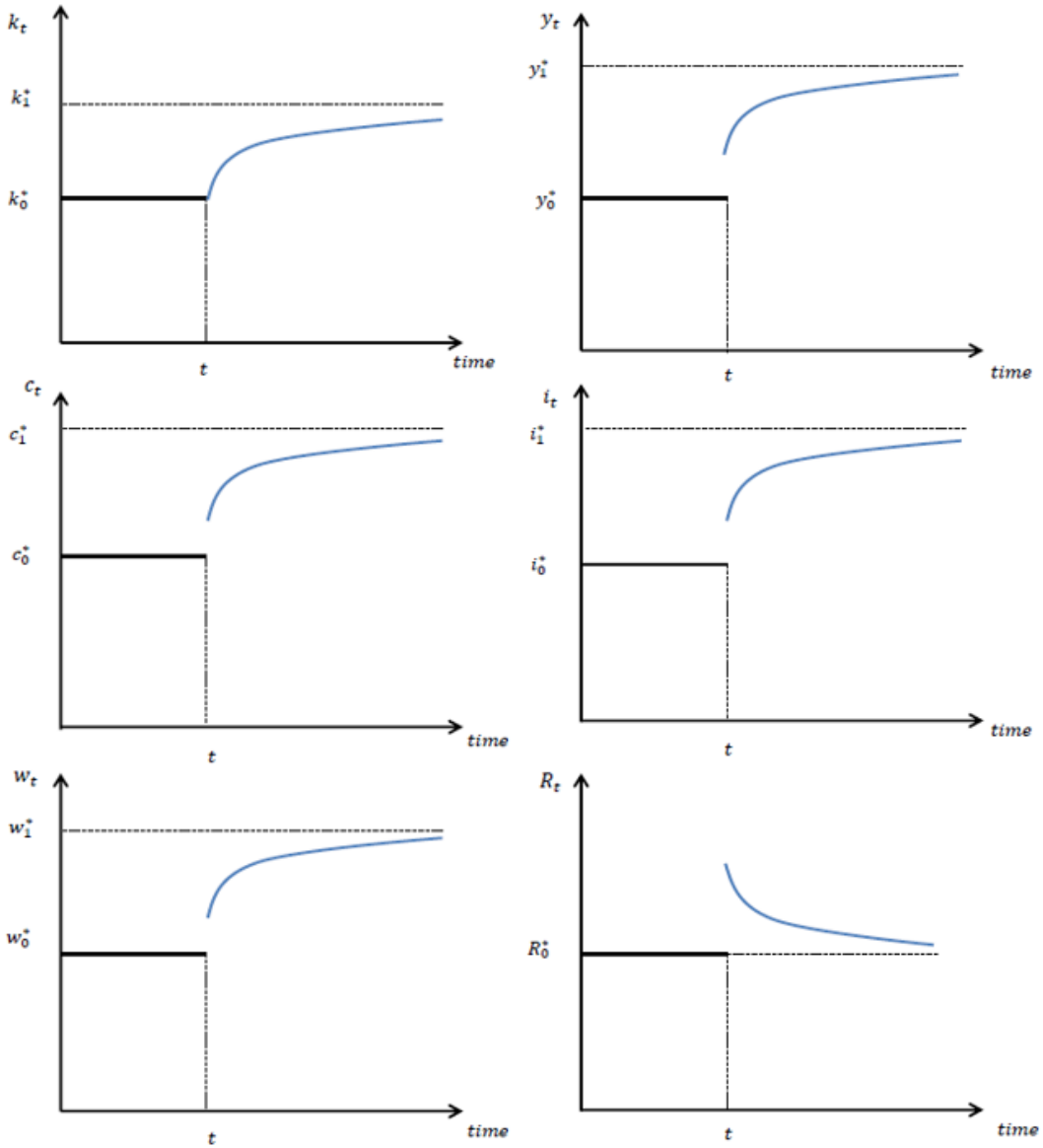
Increase in productivity

Permanent increase in A:

Figure 5.9: Exogenous Increase in A, $A_1 > A_0$



Increase in productivity



Increase in productivity

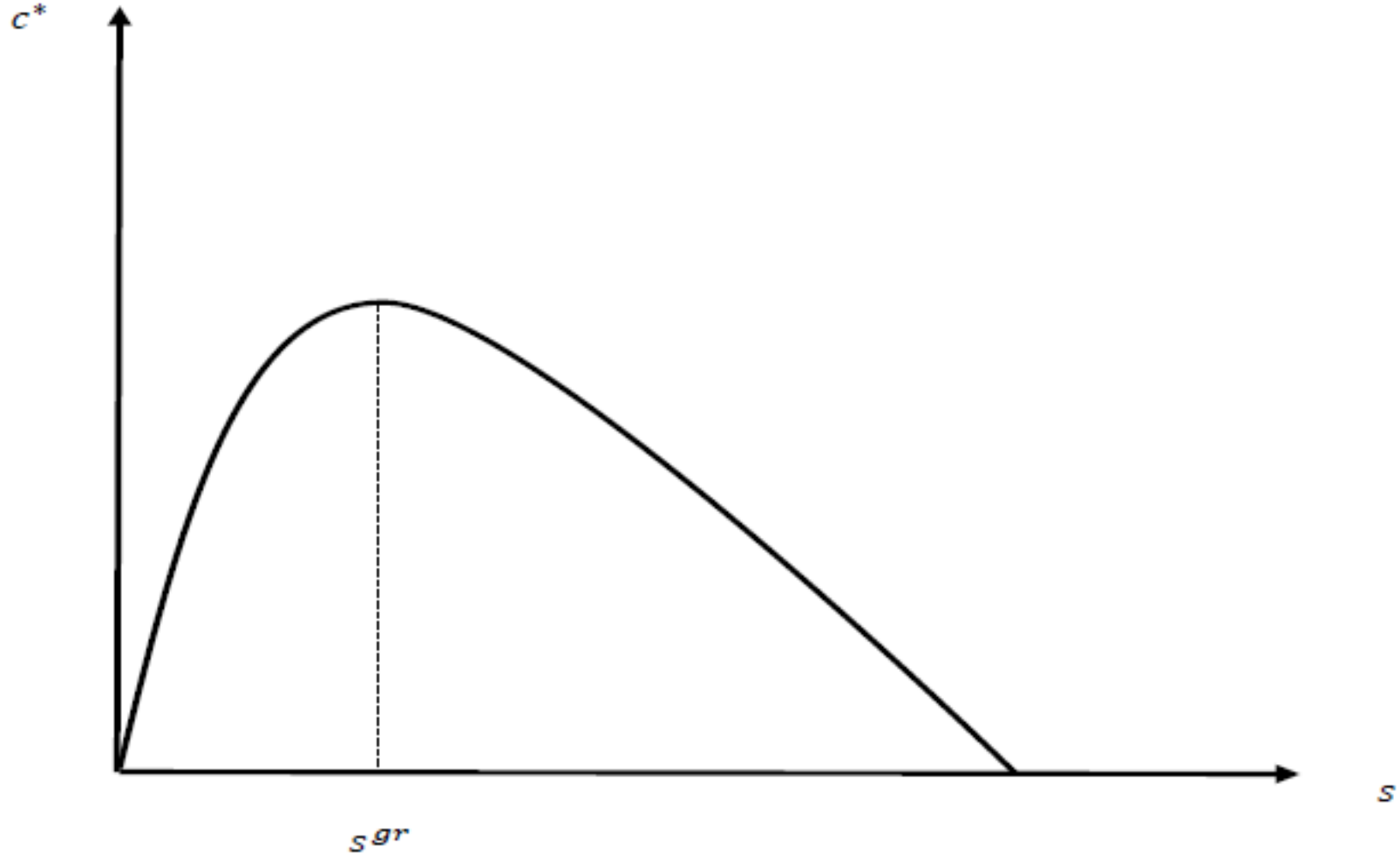
- Why no impact on R^*?

$$R^* = \alpha A(k^*)^{\alpha-1} = \alpha A \left(\frac{sA}{\delta}\right)^{\frac{\alpha-1}{1-\alpha}} = \frac{\alpha\delta}{s}$$

- This also shows why a permanent rise in s lowers R^* permanently.
- A related question: whats leel of s would maximise c^*?

The Golden Rule

Relationship between c^* and s: $c^* = (1-s)Af(k^*)$. As $s \uparrow$, $c^* \downarrow$ and $k^* \uparrow$. But as $k^* \uparrow$, $f(k^*) \uparrow$ and thus $c^* \uparrow$! What does relationship look like?



The Golden Rule

- How do we find the optimum level of saving?
- First, using the steady state result $sAf(k^*) = \delta k^*$, we find out the relationship between change in capital (dk^*) and change in saving (ds).
- Then, we use the definition of steady state consumption, $c^* = (1 - s)Af(k^*)$, and find out how consumption changes when capital changes (dk^*) and saving changes (ds).
- Then we use the result that changes in saving also changes capital, and arrive at our result.

The Golden Rule

- Golden rule: $Af'(k^*) = \delta$.
- Intuitively, what happens if $Af'(k^*) > \delta$? Change in output $>$ change in investment $\Rightarrow$ Consumption increases, which means current savings not optimum.
- What happens if $Af'(k^*) < \delta$? Change in output $<$ change in investment $\Rightarrow$ Consumption decreases, which means current savings not optimum.
- Can be shown for current model and parameter values that this condition is satisfied when $\alpha = s$.

The Augmented Solow Model[3]

- So far, population and productivity constant. So, no growth in per capital variables.
- Allow population and productivity to grow: per capital variables will grow, but variables *per efficiency unit of labour* will be constant in steady state.
- Production function:

$$Y_t = AF(K_t, Z_t N_t) \tag{1}$$

- Per efficiency unit of labour $\Rightarrow$ divide by $Z_t N_t$ instead of just N_t.
- Population growth: $N_{t+1} = (1 + n)N_t$.
- Technology growth: $Z_{t+1} = (1 + z)Z_t$.

[3] Chapter 6 of GLS.

The Augmented Solow Model

The equations of the model:

$$Y_t = AF(K_t, Z_t N_t) \tag{2}$$
$$Y_t = C_t + I_t \tag{3}$$
$$K_{t+1} = I_t + (1-\delta)K_t \tag{4}$$
$$I_t = sY_t \tag{5}$$
$$w_t = AF_N(K_t, Z_t N_t) \tag{6}$$
$$R_t = AF_K(K_t, Z_t N_t) \tag{7}$$
$$Z_t = (1+z)^t \tag{8}$$
$$N_t = (1+n)^t. \tag{9}$$

The Augmented Solow Model

- The capital accumulation equation:

$$K_{t+1} = sAF(K_t, Z_t N_t) + (1 - \delta)K_t$$

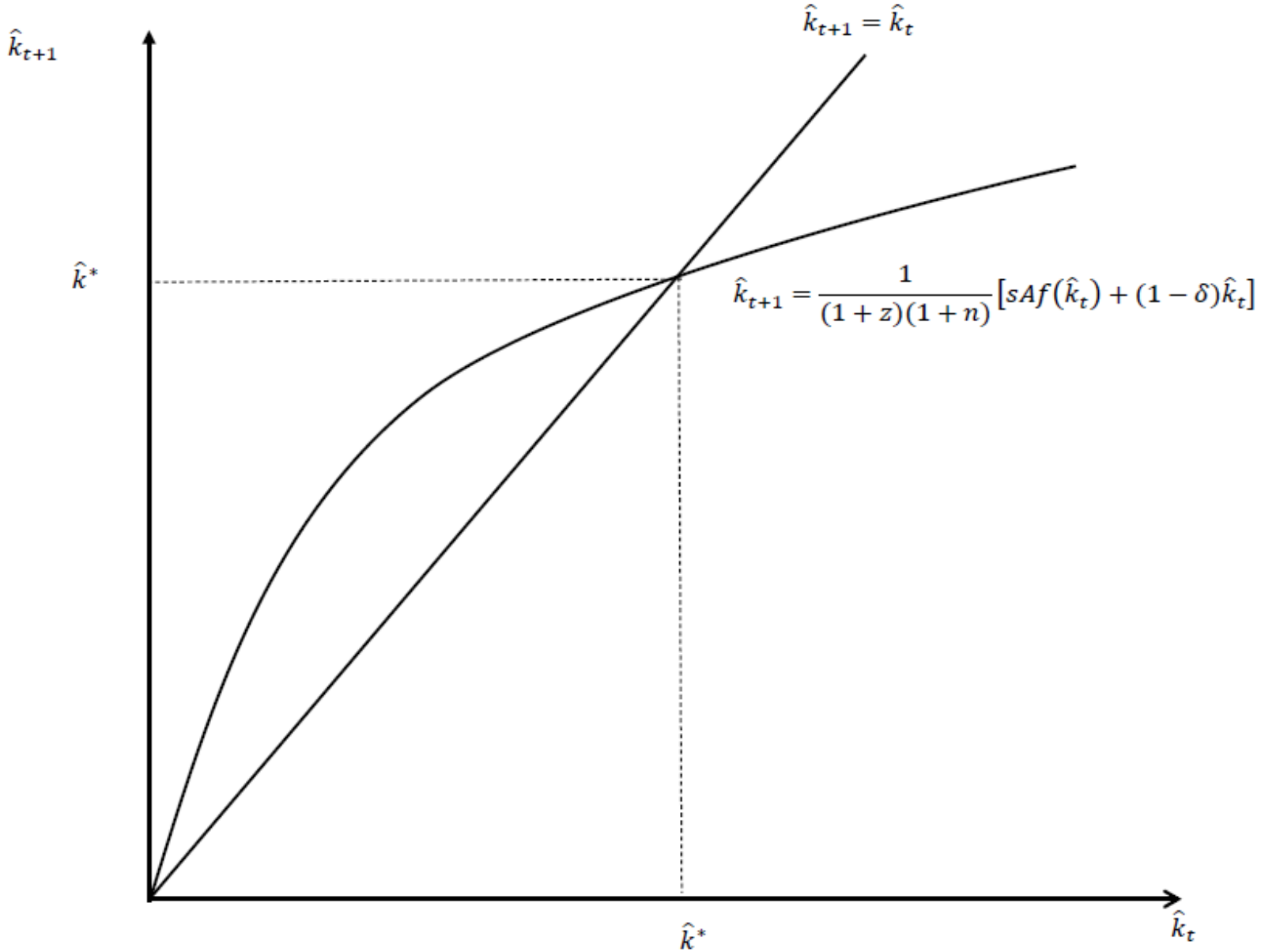
- Similar to the basic model, we now divide by $Z_t N_t$ to arrive at:

$$\hat{k}_{t+1} = \frac{1}{(1+n)(1+z)} \left[sAf(\hat{k}_t) + (1 - \delta)\hat{k}_t \right]. \tag{10}$$

- Similarly, we will convert all variables to per effective unit of labour terms, or divide through by $Z_t N_t$:

$$\begin{aligned}
\hat{y}_t &= Af(\hat{k}_t) \\
\hat{c}_t &= (1 - s)Af(\hat{k}_t) \\
\hat{i}_t &= sAf(\hat{k}_t) \\
R_t &= Af'(\hat{k}_t) \\
w_t &= Z_t[Af(\hat{k}_t) - Af'(\hat{k}_t)\hat{k}_t].
\end{aligned}$$

The Augmented Solow Model



The Augmented Solow Model

- In steady state, $\hat{x}_t = \hat{x}_{t+1} = \hat{x}^*$ for all variables.
- What about per capital variables?

$$\begin{aligned} \frac{K_{t+1}}{Z_{t+1}N_{t+1}} &= \frac{K_t}{Z_t N_t} \\ \frac{K_{t+1}}{N_{t+1}} &= \frac{K_t}{N_t}\frac{Z_{t+1}}{Z_t} \\ k_{t+1} &= k_t(1+z) \\ \frac{k_{t+1}}{k_t} &= (1+z) \end{aligned} \tag{11}$$

- Per capital variables growing at the rate of $(1+z)$ even in steady state.
- Similarly, variables in levels growing at the rate of $(1+n)(1+z)$ in the steady state.

The Augmented Solow Model

How well does the model perform/ Check Kaldor facts:

- Output per worker grows at a (roughly) constant rate: Yes.
- Capital per worker grows at a (roughly) constant rate: Yes.
- Capital-to-output ratio is (roughly) constant: Yes.
- Labour's share of income is (roughly) constant: Yes (implied in the Cobb-Douglas function).
- Rate of return on capital is (roughly) constant: Yes.
- Real wages grow at a sustained, approximately constant rate: Yes, at the rate of growth of Z_t.

Productivity and population growth in the model helps us match data facts.

Exercise

- Model equations:

$$\hat{k}_{t+1} = \frac{1}{(1+n)(1+z)}\left[sA\hat{k}_t^{\alpha} + (1-\delta)\hat{k}_t^{\alpha}\right]$$
$$\hat{y}_t = A\hat{k}_t^{\alpha}$$
$$\hat{c}_t = (1-s)A\hat{k}_t^{\alpha}$$
$$\hat{i}_t = sA\hat{k}_t^{\alpha}$$
$$R_t = \alpha A\hat{k}_t^{\alpha-1}$$
$$w_t = (1-\alpha)Z_t A\hat{k}_t^{\alpha}.$$

- Capital steady state:

$$\hat{k}^* = \left[\frac{sA}{(1+z)(1+n)-(1-\delta)}\right]^{\frac{1}{1-\alpha}} \tag{12}$$

- $s = 0.2$, $A = 1$, $\delta = 0.1$, $\alpha = 0.33$, $z = 0.02$, and $n = 0.01$.

- From $t = 0$ till $t = 8$, economy stays in steady state. Then from $t = 9$, $s = 0.3$. Solve for the transitions and plot results!

End