Economic Theory II
Lecture 2 [1]

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[1] Content of current lecture slides derives on slides prepared by Dr Valentin Jouvanceau previously.

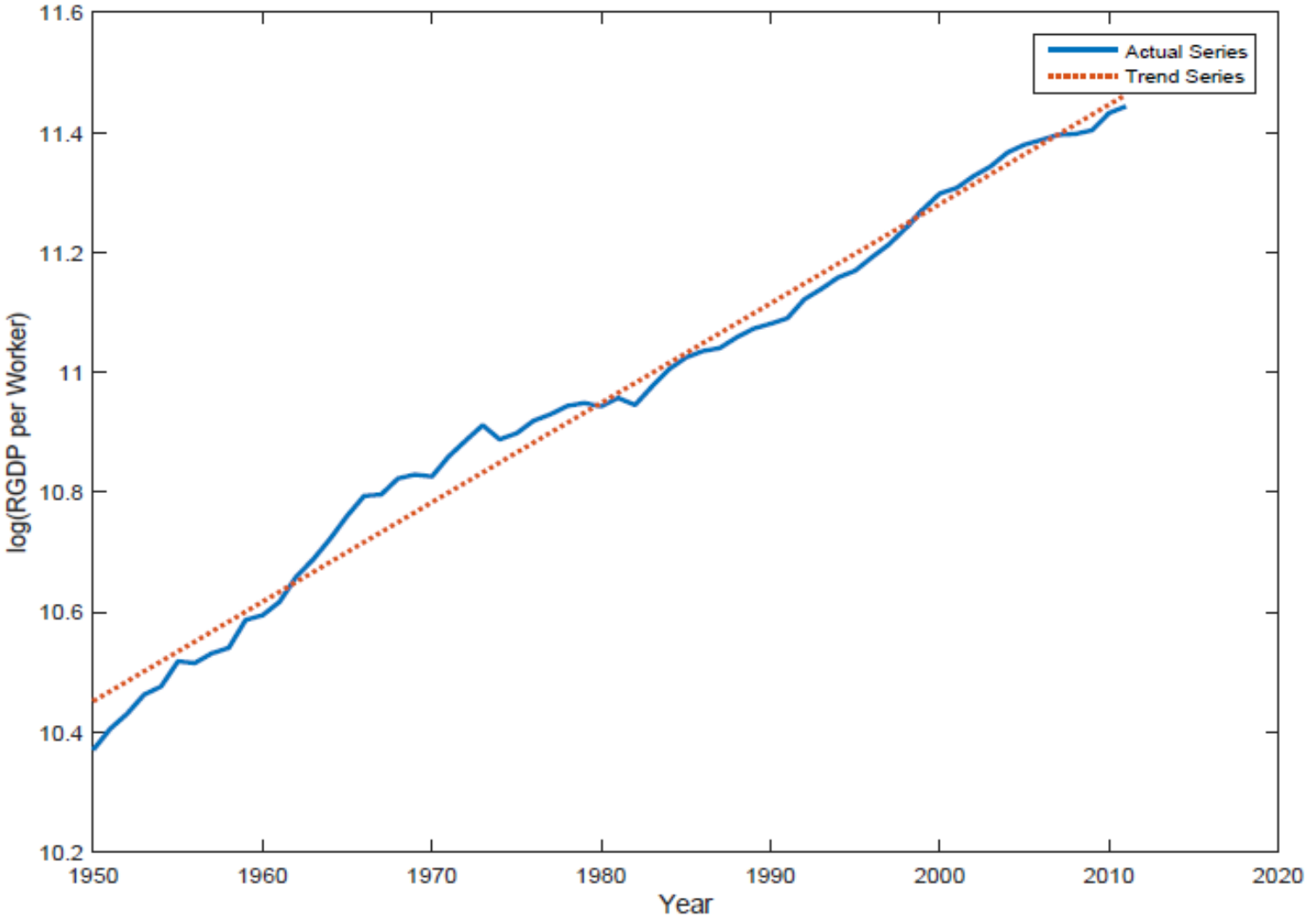
The Long Run [2]

- What are the characteristics of an economy in the long run? How do variables behave?
- What are the determinants of long run growth?
- 'Growth' usually refers to rate of increase in *Real GDP per capita*.
- Nicholas Kaldor, in 1957, listed some "stylised facts" from data.
- Came to be known as "Kaldor Facts", which continue to provide a reasonably accurate description of economic growth across developed countries, including USA.

[2] Chapter 4 of GLS.

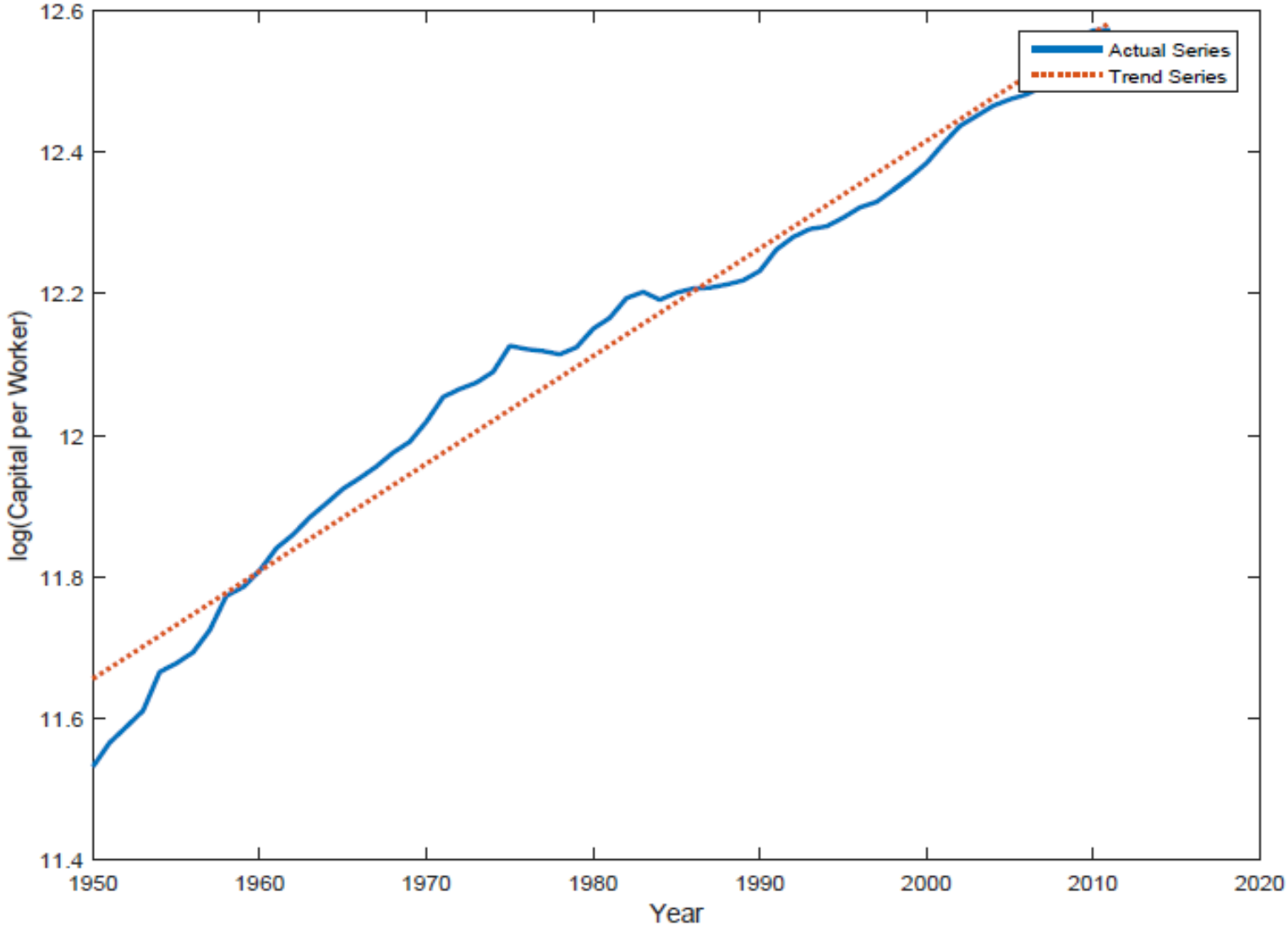
Kaldor Facts

1. Output per worker grows at a (roughly) constant rate.



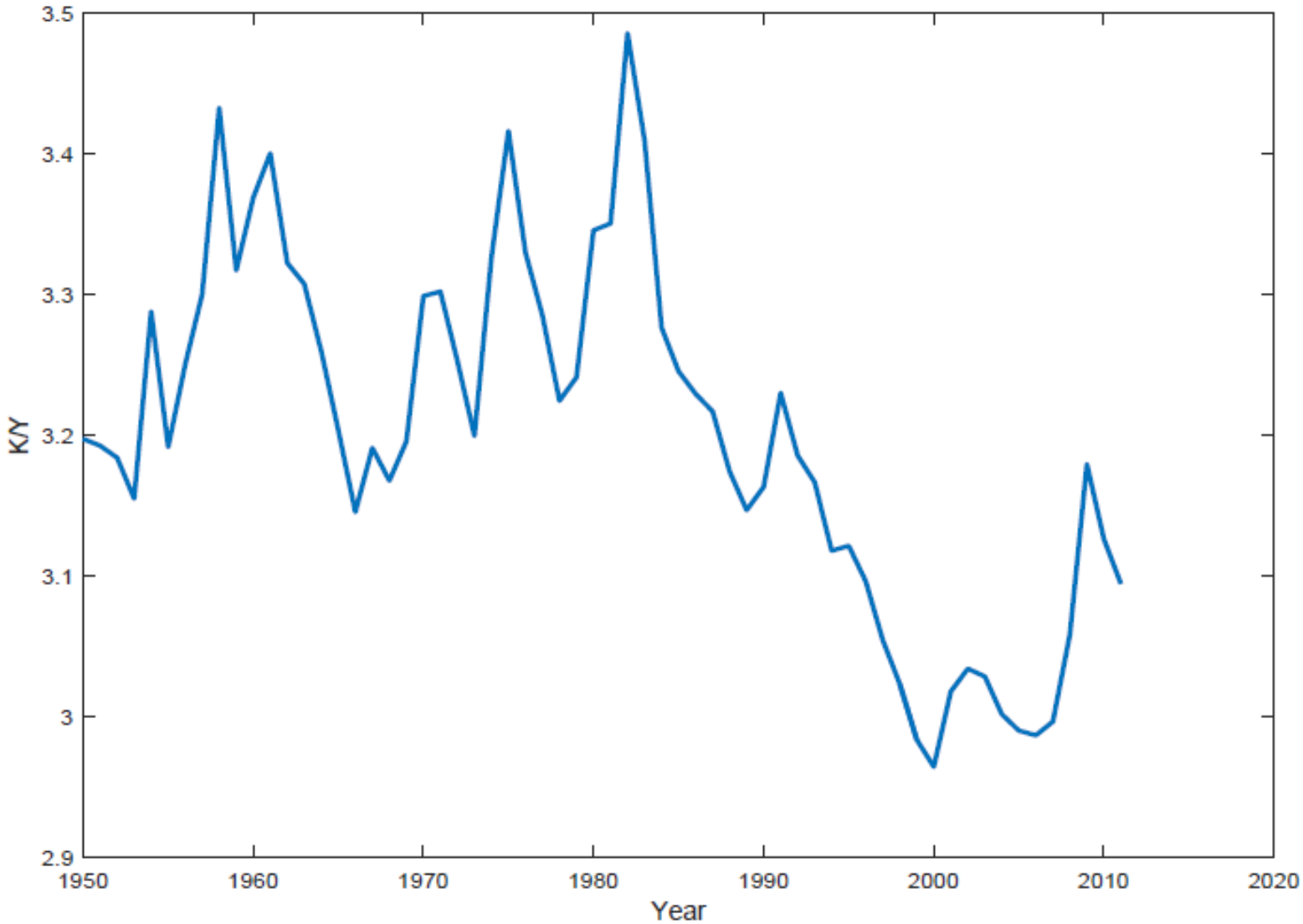
Kaldor Facts

2. Capital per worker grows at a (roughly) constant rate.



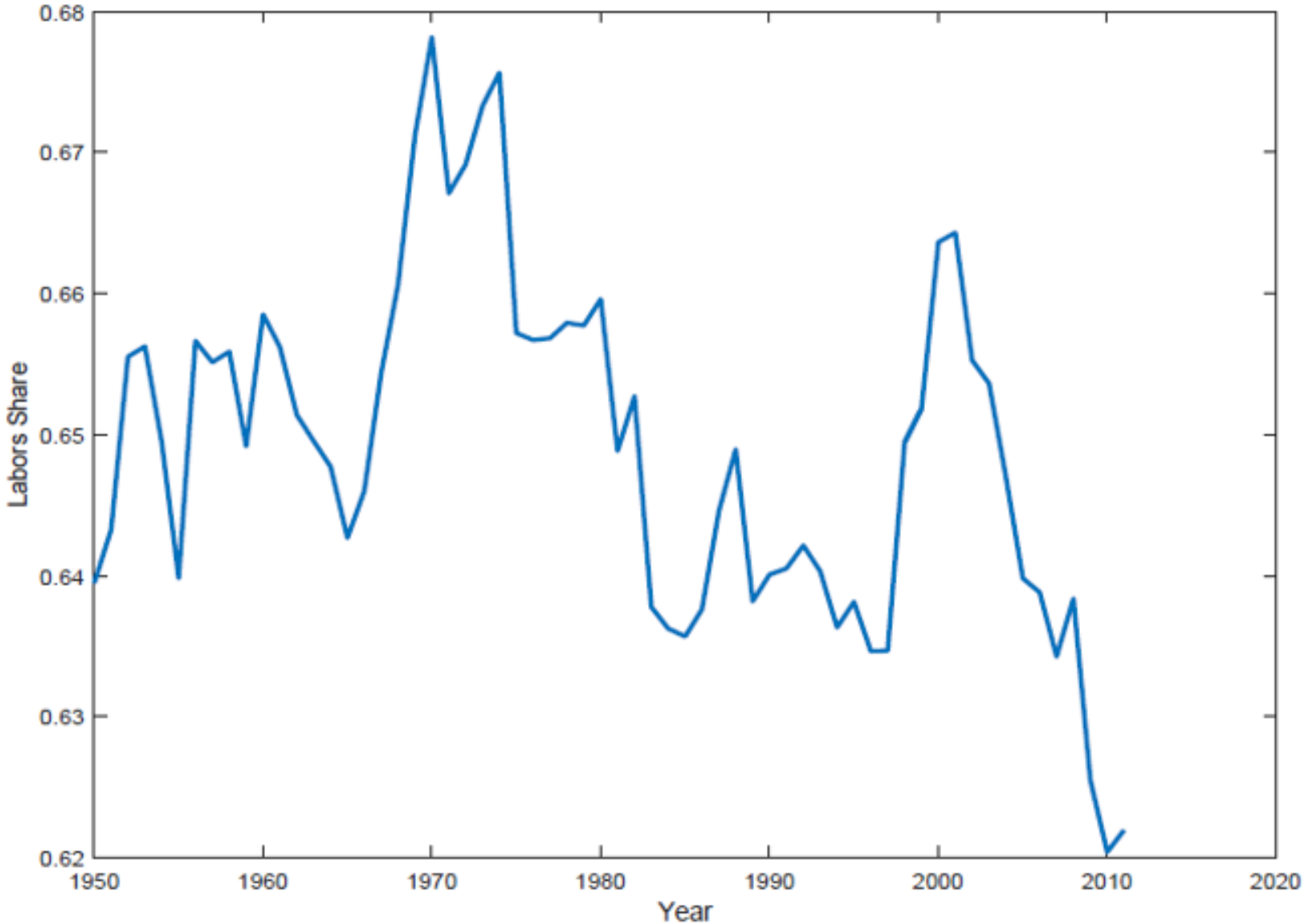
Kaldor Facts

3. Capital-to-output ratio is (roughly) constant.



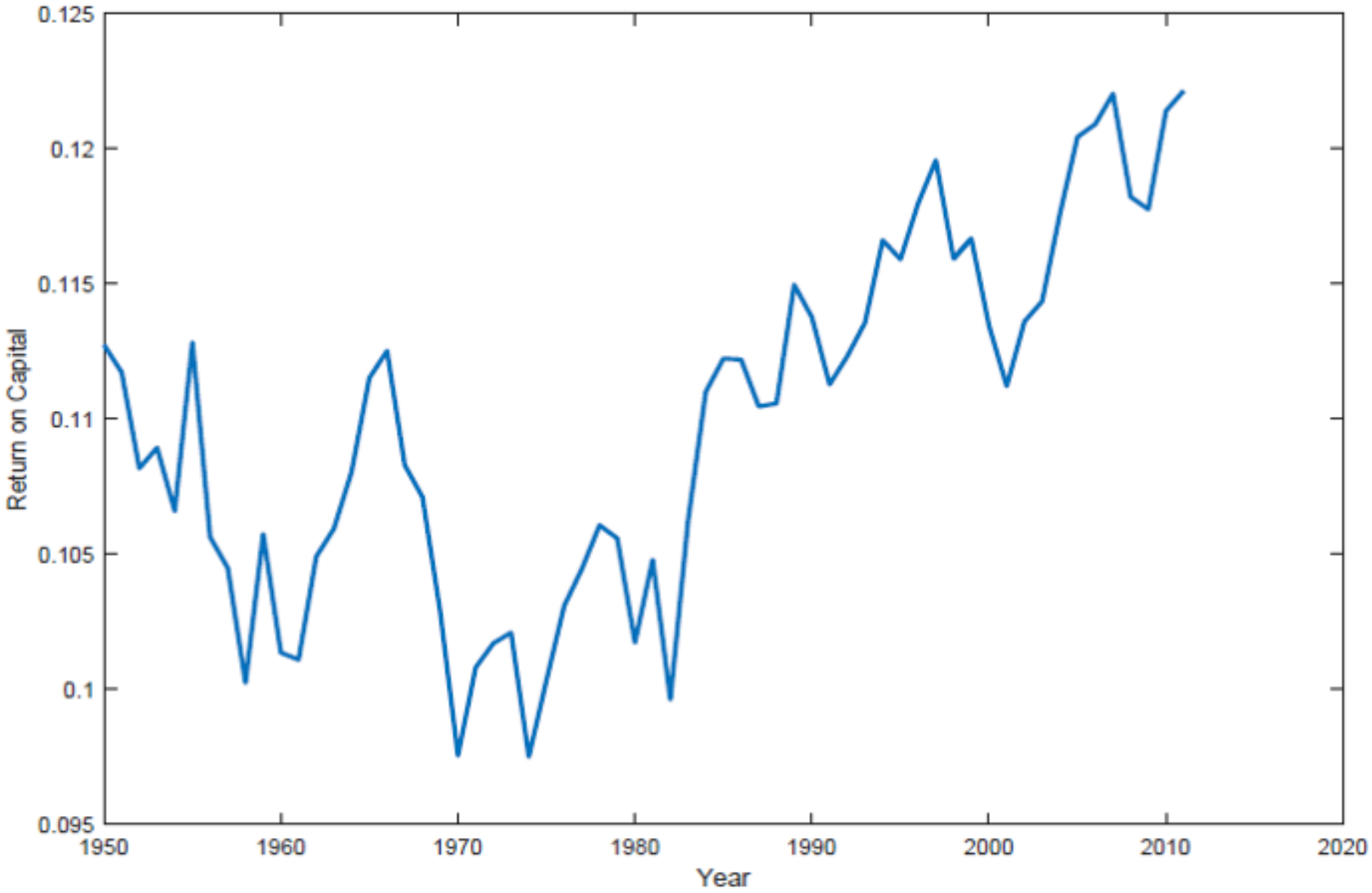
Kaldor Facts

4. Labour's share of income $LABSH_t = \frac{w_t N_t}{Y_t}$ is (roughly) constant.



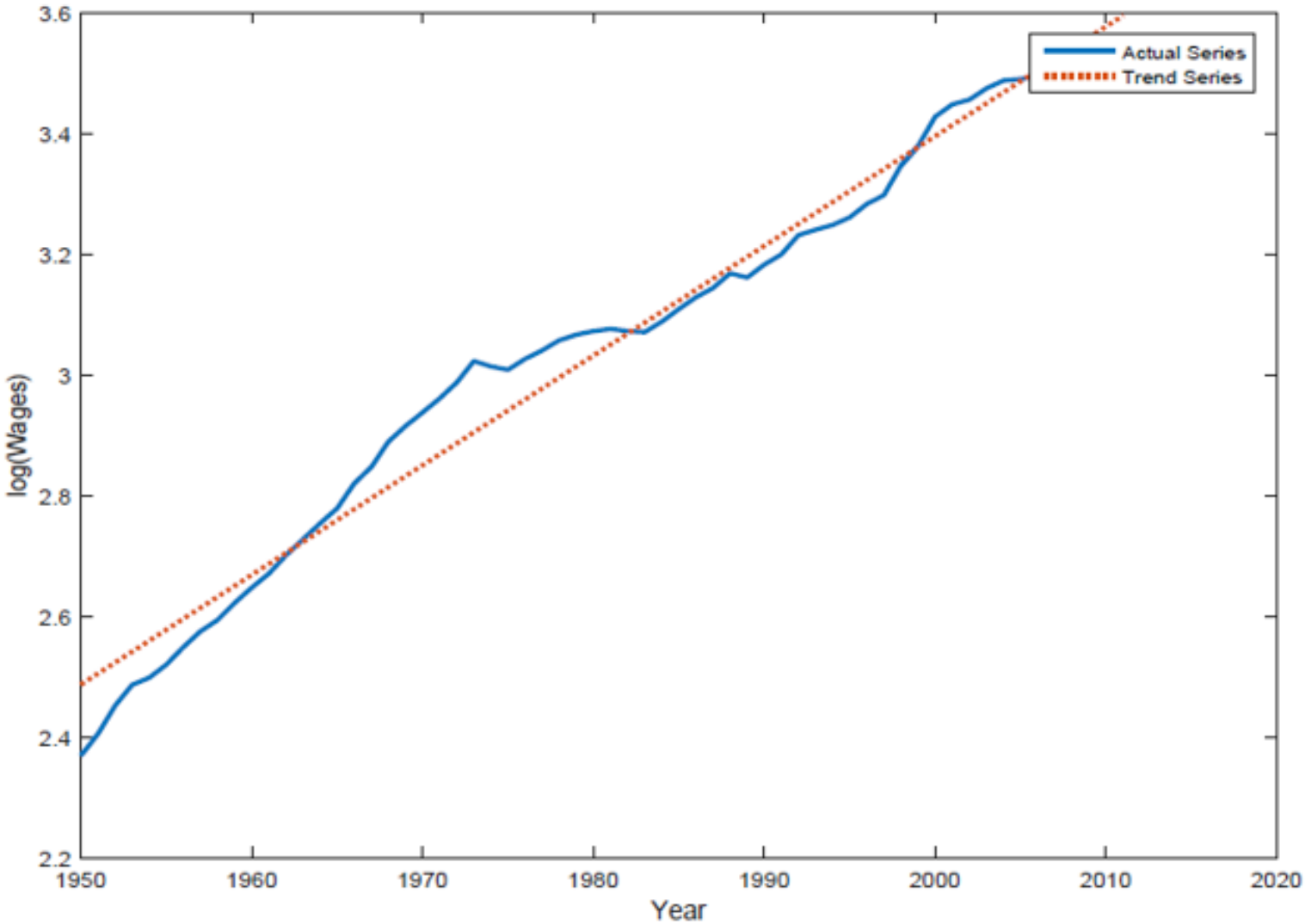
Kaldor Facts

5. Rate of return on capital $R_t = CAPSH_t \times \frac{Y_t}{K_t}$ is (roughly) constant.



Kaldor Facts

6. Real wages grow at a sustained, approximately constant rate.



Cross country facts

1. Enormous variations in income across countries. (2011 figures)

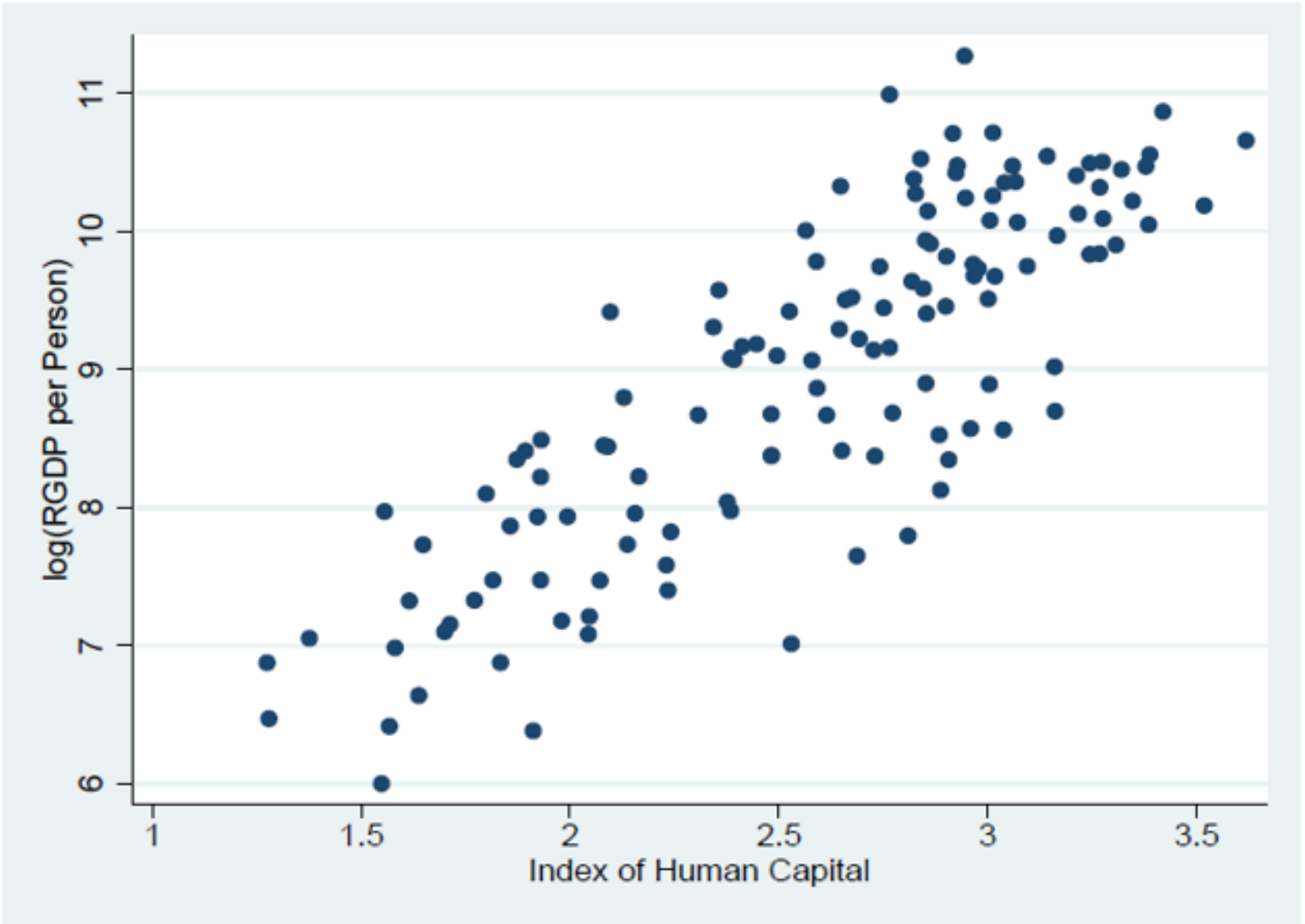
		GDP per Person
High income countries		
	Canada	$35,180
	Germany	$34,383
	Japan	$30,232
	Singapore	$59,149
	United Kingdom	$32,116
	United States	$42,426
Middle income countries		
	China	$8,640
	Dominican Republic	$8,694
	Mexico	$12,648
	South Africa	$10,831
	Thailand	$9,567
	Uruguay	$13,388
Low income countries		
	Cambodia	$2,607
	Chad	$2,350
	India	$3,719
	Kenya	$1636
	Mali	$1,157
	Nepal	$1,281

Notes: This data comes from the Penn World Tables, version 8.1. The real GDP is in terms of chain-weighted PPPs.

LT $\sim \$23,600$ (2011).

Cross country facts

2. Strong, positive correlation between income per capita and "human capital", or, roughly, years of education. (Implied causation...?)



The Basic Solow Model [3]

- We see how the Solow model can generate Kaldor Facts, and maybe explain some dynamics of cross country differences as well.
- Before going to the complete Solow model, we will understand some concepts, mechanics, and algebra of the model in a simplified setting.
- Following simplifications are considered in the Basic Solow Model:
  - Population is constant.
  - Technology (or Productivity) is constant.
- These simplifications mean that, in this simplified version, there is no growth.
- Main purpose is to fix concepts.

[3] Chapter 5 of GLS.

The Basic Solow Model

- Essential mathematical concepts:
  - Growth rates.
  - Calculus.
  - Homogeneity of functions.
  - Returns to scale.
  - Euler's theorem for homogeneous functions.
  - Total differential.
  - Optimisation.

The Basic Solow Model

Aggregate production:

- Inputs into production process are 'capital' and 'labour'.
- Capital:
  - Must itself be produced.
  - Durable, hence a stock concept. Can be accumulated.
- Labour:
  - Is an endowment (hours in a day).
  - Flow concept, cannot be accumulated.
- $Y_t = AF(K_t, N_t)$. A is constant productivity.

The Basic Solow Model

- Properties of the production function:
  1. Both inputs necessary: $F(K,0) = F(0,N) = 0$.
  2. Marginal products positive: $F_K > 0$ and $F_N > 0$.
  3. Diminishing marginal products: $F_{KK} < 0$ and $F_{NN} < 0$.
  4. Increase in one factor increases marginal product of other factor: $F_{KN} = F_{NK} > 0$.
  5. Constant returns to scale. Doubling both inputs doubles output: $F(\gamma K, \gamma N) = \gamma^{\rho} F(K,N)$, where $\rho = 1$.

- A functional form which satisfies all properties is 'Cobb-Douglas':

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha} \tag{1}$$

The Basic Solow Model

- Firms maximise profits as under:

$$\max_{K,N} \Pi_t = AF(K_t, N_t) - w_t N_t - R_t K_t$$

.

- Optimisation shows what the wages w and rental rate of capital R should be:

$$w_t = A_t F_N(K_t, N_t)$$
$$R_t = A_t F_K(K_t, N_t).$$

- Revenue, wages, and rental rate stated in 'real' terms, not in 'nominal' or 'money' terms (i.e. in terms of the units of real output).

The Basic Solow Model

Households:

- Household budget constraint is:

$$C_t + I_t \leq w_t N_t + R_t K_t + \Pi_t$$

.

- Profits Π are 0 in perfect competition, which implies that $w_t N_t + R_t K_t = Y_t$ from the firm problem.

- Using this in the household budget constraint:

$$C_t + I_t = Y_t$$

.

- This is the goods market clearing condition, also known as the national income accounting identity.

The Basic Solow Model

- Capital accumulation:

$$K_{t+1} = I_t + (1 - \delta)K_t$$

.

- Investment is a constant fraction of output: $I_t = sY_t$.
- So: $C_t + sY_t = Y_t \Rightarrow C_t = (1 - s)Y_t$.
- Constant saving rate, although not optimal from microeconomic perspective, fits data well over long periods of time.
- Finally, given our simplification regarding population being constant, we assume that the labour supply N is constant: $N_t = N \ \ \forall t \in [0, \infty)$.

The Basic Solow Model

The Basic Solow Model looks like this:

$$Y_t = AF(K_t, N_t) \tag{2}$$
$$Y_t = C_t + I_t \tag{3}$$
$$K_{t+1} = I_t + (1 - \delta)K_t \tag{4}$$
$$I_t = sY_t \tag{5}$$
$$w_t = AF_N(K_t, N_t) \tag{6}$$
$$R_t = AF_K(K_t, N_t). \tag{7}$$

6 equations in 6 variables: Y , C , K , N , I , w , R. Exogenous variable A and parameters s and δ.

The Basic Solow Model

- Combining (2), (4), and (5), and dividing through by N_t, we have:

$$\frac{K_{t+1}}{N_t} = \frac{sAF(K_t, N_t)}{N_t} + (1-\delta)\frac{K_t}{N_t}. \tag{8}$$

- But we know that $F(.)$ is constant returns to scale. So, dividing $F(.)$ by something is equivalent to dividing each input by that amount[4]:

$$\frac{F(K_t, N_t)}{N_t} = F\left(\frac{K_t}{N_t}, \frac{N_t}{N_t}\right) = F(k_t, 1)$$

where we define $k_t := \frac{K_t}{N_t}$, and $f(k_t) := F(k_t, 1)$.

[4] Technically, this property results from the function being linearly homogeneous. Refer Alpha Chiang's book, pages 410-418.

The Basic Solow Model

- Now, (8) becomes:

$$\frac{K_{t+1}}{N_t} = sAf(k_t) + (1-\delta)k_t$$

.

- Multiplying and dividing the left hand side by N_{t+1}, rearranging, and using the fact that $N_{t+1} = N_t = N$, we get:

$$\frac{K_{t+1}}{N_{t+1}}\frac{N_{t+1}}{N_t} \equiv k_{t+1} = sAf(k_t) + (1-\delta)k_t. \tag{9}$$

- Similarly, we will convert all variables to per capital terms, or divide through by N_t:

$$\begin{aligned} y_t &= Af(k_t) \\ c_t &= (1-s)Af(k_t) \\ i_t &= sAf(k_t). \end{aligned}$$

The Basic Solow Model

- What about R_t in (6) and w_t in (7)? We use Euler's theorem.
- Euler:
  1. If $\rho = 1$ in $F(\gamma K, \gamma N) = \gamma^{\rho} F(K, N)$, then;

$$F = F_K + F_N N \tag{10}$$

  2. If $\rho = 1$ in $F(\gamma K, \gamma N) = \gamma^{\rho} F(K, N)$, then;

$$F_K(K_t, N_t) = F\left(\frac{K_t}{N_t}, \frac{N_t}{N_t}\right) = f'(k_t) \tag{11}$$

The Basic Solow Model

- From (10), we can directly get that

$$R_t = Af'(k_t). \tag{12}$$

- From (11) and (12);

$$\begin{aligned} F(K_t, N_t) &= f'(k_t)K_t + F_N(K_t, N_t)N_t \\ f(k_t) &= f'(k_t)k_t + F_N(K_t, N_t) \\ F_N(K_t, N_t) &= f(k_t) - f'(k_t)k_t \\ \Rightarrow w_t &= f(k_t) - f'(k_t)k_t \end{aligned} \tag{13}$$

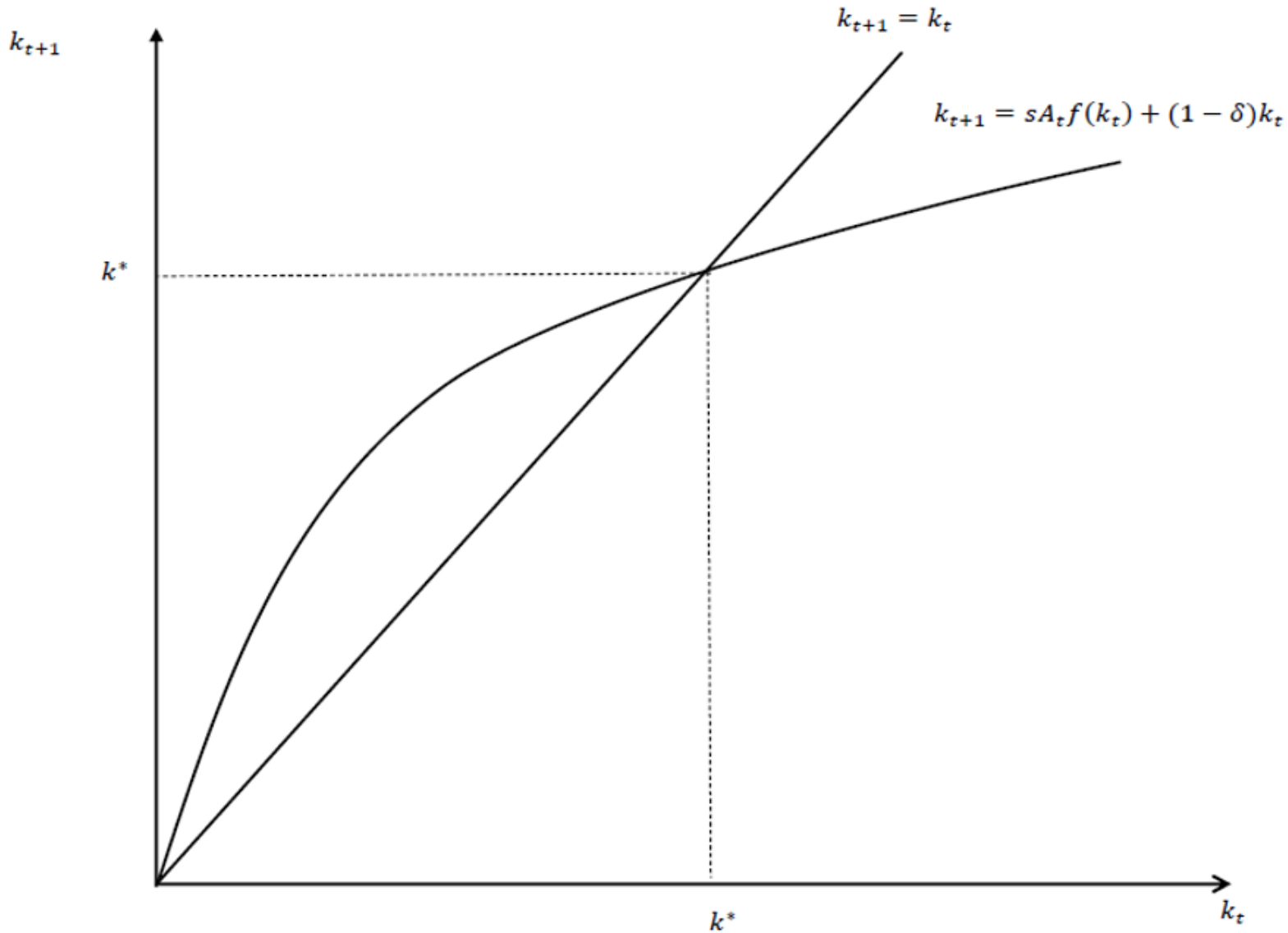
The Basic Solow Model

If we consider the Cobb-Douglas production function in (1), we have the following system;

$$y_t = Ak_t^{\alpha} \tag{14}$$
$$c_t = (1-s)Ak_t^{\alpha} \tag{15}$$
$$i_t = sAk_t^{\alpha} \tag{16}$$
$$R_t = \alpha Ak_t^{\alpha-1} \tag{17}$$
$$w_t = Ak_t^{\alpha} - \alpha Ak_t^{\alpha-1}k_t = (1-\alpha)Ak_t^{\alpha} \tag{18}$$
$$k_{t+1} = sAk_t^{\alpha} + (1-\delta)k_t \tag{19}$$

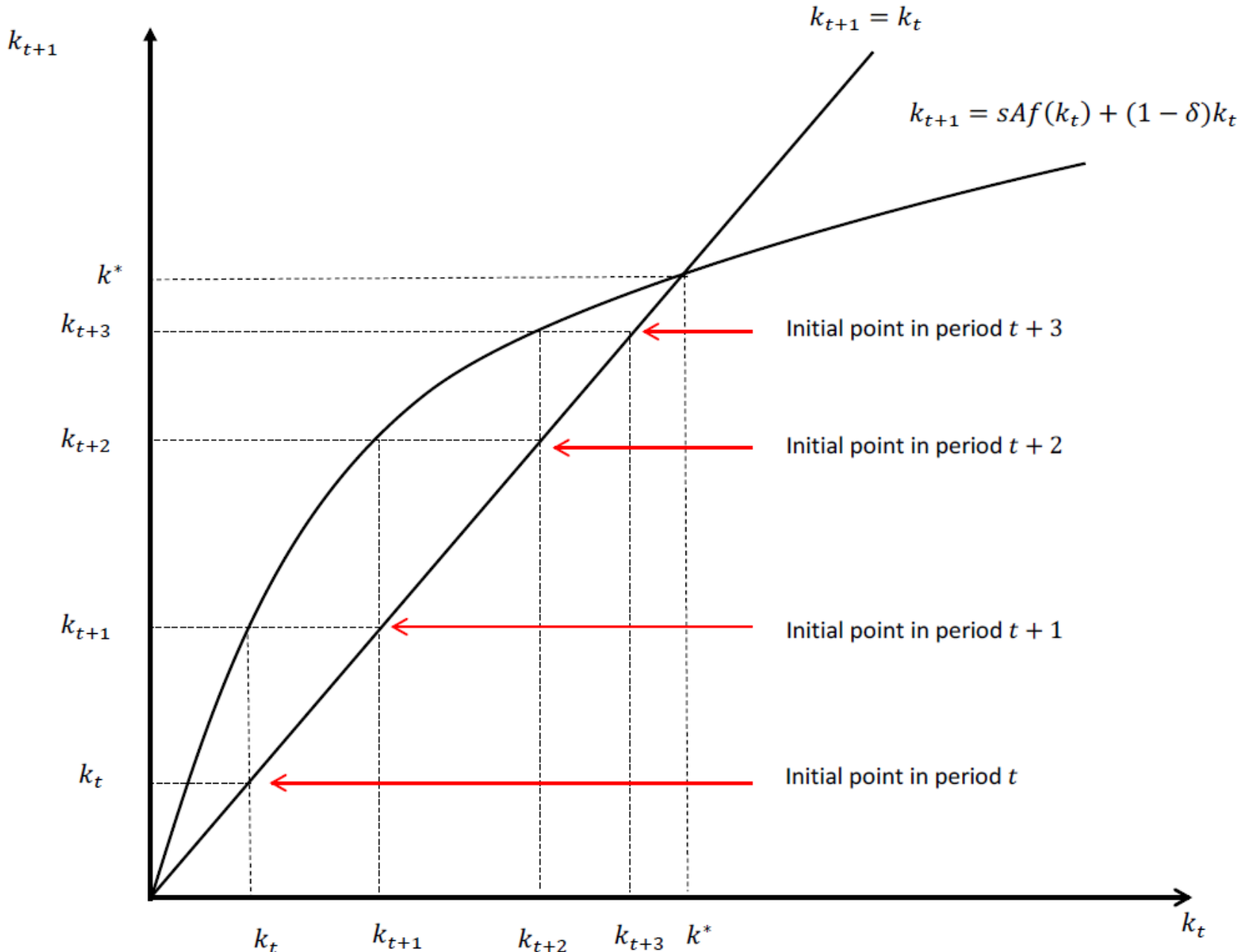
Graphical Analysis

We graph the (per capital) capital evolution. k^* is the 'steady state': once economy reaches this, per capita variables are constant.



Graphical Analysis

Dynamics from below (and from above) k^*:



The Steady State

- In the steady state, $k_{t+1} = k_t = k^*$.
- The system is now as under:

$$y^* = A(k^*)^\alpha \tag{20}$$
$$c^* = (1-s)A(k^*)^\alpha \tag{21}$$
$$i^* = sA(k^*)^\alpha \tag{22}$$
$$R^* = \alpha A(k^*)^{\alpha-1} \tag{23}$$
$$w* = A(k^*)^\alpha - \alpha A(k^*)^{\alpha-1}k^* = (1-\alpha)A(k^*)^\alpha \tag{24}$$
$$k^* = sA(k^*)^\alpha + (1-\delta)k^* \tag{25}$$

- k^* pinned down, so everything else pinned down.

Recap

- Firms maximise profits by choosing capital and labour inputs.
- Rent and wages paid to households are used for consumption and investment.
- Investment adds to the stock of capital for production in future periods.
- Investment (and, as a result, consumption as well) assumed to be a constant fraction of total output.
- We convert model to *per capita* terms to make it comparable to Kaldor facts.
- In the absence of population and productivity growth, per capita variables reach a stationary point, and are constant perpetually thereafter.

Next...

We conduct some experiments to check what the impact of a one-time increase in saving rate s, or productivity A, has on the model.

End